Rank Reversal and the Productivity Losses from Resource Misallocation

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In efficient economies, firm size and productivity rankings coincide. Using firm-level data from 17 countries, we demonstrate that distortions reverse this ordering, yielding size-productivity rank correlations ranging from 0.75 in advanced economies to 0.25 in developing economies. We construct counterfactuals where idiosyncratic distortions are productivity dependent but never reverse rankings. In these economies, the average *TFP* gain from eliminating misallocation decreases from 50% to 14%, implying that most aggregate losses stem from rank reversals. Finally, we consider state-owned enterprises and financial frictions as drivers of rank reversal. We find the latter to be a more promising source.

Keywords: misallocation, rank reversal, productivity

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1. Introduction

A defining feature of efficient economies is that the most productive firms are also the largest, resulting in a perfect correlation between firm size and productivity rankings. In many countries, however, allocative distortions—such as financial frictions, size-dependent policies, or subsidized state-owned enterprises—impede this process, allowing less productive firms to operate at larger scales than their more productive counterparts. We refer to such outcomes as rank reversal. The goal of this paper is to conduct a systematic cross-country analysis of the prevalence of rank reversal and quantify its contribution to the aggregate productivity gains resulting from the removal of resource misallocation.

Using firm-level data from 17 countries, we demonstrate that rank reversal is widespread, strongly correlated with income levels, and accounts for a significant share of the overall losses from resource misallocation. The rank correlation between firm size and productivity ranges from 0.75 in more developed economies to 0.25 in less developed ones. To assess its quantitative importance, we construct counterfactual economies where distortions preserve their systematic productivity-dependence but never overturn firm rankings. In these economies, the average gains from removing misallocation fall sharply—from 50% to just 14%—suggesting that rank reversal accounts for the bulk of aggregate *TFP* losses. Last, we study the extent of rank reversal in economies with collateral constraints and among state-owned enterprises. While we document substantial rank reversal among SOEs, we find financial frictions to be a more promising source of the rank reversal in the data.

Our framework builds on Hsieh and Klenow (2009), in which misallocation is reflected in the dispersion of marginal products across firms.[1] In this setting, an efficient allocation implies a perfect rank correlation between size and productivity. Distortions may break this link, so the extent of rank reversal can be measured by how far the correlation between firm size and productivity rankings falls below one. As stated, we find that the rank correlation ranges from 0.75 to 0.25 between the most and least developed countries in our sample.

We then investigate the underlying sources of rank reversal by analyzing how closely firm-level distortions (*TFPR*) are related to physical productivity (*TFPQ*). For rank reversal to occur, a less productive firm must receive a sufficiently large implicit subsidy—and a more productive one a sufficiently large implicit tax—so that the size rankings implied by their productivities are reversed. Therefore, an informative statistic for the average degree of alignment between distortions and productivity is the elasticity of distortions

[1] The model in Hsieh and Klenow (2009) features a continuum of heterogeneous firms producing differentiated varieties under CES demand. Firms hire capital and labor in competitive markets, and factors can be reallocated costlessly. Misallocation is characterized by dispersion in marginal revenue products, which would be equalized across firms in an efficient allocation.

with respect to productivity.

We estimate the productivity-elasticity of idiosyncratic distortions by regressing the log of demeaned firm-level distortions on the log of demeaned productivity. Across all countries in our sample, we find positive elasticities, indicating that more productive firms, on average, face higher distortions. However, these elasticities are uniformly below the threshold required to reverse firm size rankings. That is, while distortions compress the size gap between more and less productive firms, they typically do not invert their order. Rank reversal, then, arises from firm-level deviations around this average pattern.

This insight motivates our strategy to isolate the contribution of rank reversal to overall misallocation. Using the estimated productivity elasticity of distortions and the observed distribution of firm productivity, we construct counterfactual productivity-dependent distortions that preserve the average productivity dependence of distortions without overturning firm rankings. By comparing the *TFP* gains from removing these counterfactual distortions with those from eliminating the actual distortions in the data, we identify an upper bound on the share of misallocation costs that can be attributed specifically to rank reversal.[2]

We find that rank reversal accounted for most of the aggregate gains from resolving misallocation. Abstracting from rank reversal leads to significantly smaller productivity gains than removing the actual distortions observed in the data. While eliminating the observed misallocation yields average TFP gains of 50%, removing the non-rank-reversing component yields average gains of just 14%. This gap underscores the pivotal role of rank reversal in magnifying the aggregate costs of misallocation.

Finally, we investigate two sources of rank reversal. First, we demonstrate that sufficiently tight financial frictions with collateral constraints, in an environment similar to Buera and Shin (2013), can overturn the size-productivity ordering, thereby rationalizing the substantial productivity costs associated with credit market imperfections. Second, we find that State-Owned Enterprises display pronounced rank reversal, but substantial reversal persists even among privately owned firms. This result suggests that SOEs exacerbate, but do not fully account for, the phenomenon, whereas financial frictions show more promise.

2. Related Literature

This paper contributes to the large literature on resource misallocation and its consequences for aggregate productivity. A seminal contribution by Restuccia and Rogerson (2008) demonstrated how policy distortions that alter firms' relative sizes can generate

[2]Our quantification delivers an upper bound in the sense that it removes all the dispersion of distortions around its systematic relationship with *TFPQ*, thus abstracting from non-rank-reversing dispersion that would still carry productivity losses.

sizable losses in total factor productivity (*TFP*). Hsieh and Klenow (2009) formalized this idea in a framework where misallocation is reflected in the dispersion of marginal revenue products across firms and proposed a strategy to infer distortions from firm-level data.[3]. Building on this framework, our paper identifies a novel statistic for characterizing the aggregate implications of idiosyncratic distortions: their implied degree of size-productivity rank reversals.

The motivation for studying rank reversals more systematically stems from Hopenhayn (2014). That paper proposes a measure of idiosyncratic distortions based on comparing the firm-size distribution in a distorted economy to that in an efficient benchmark. They find that this measure implies only modest aggregate productivity losses, largely because it does not generate rank reversals between firm productivity and size. Building on this insight, our work expands the assessment of rank reversal as a statistic that captures the macroeconomic consequences of idiosyncratic distortions. We extend the analysis across a broad cross-country dataset, develop a strategy to quantify the contribution of rank reversal to aggregate *TFP* losses, and characterize its manifestation in two specific sources of misallocation: state-owned enterprises and financial frictions.

Our strategy to isolate the contribution of rank reversal also builds on the subset of the literature emphasizing the productivity-dependent nature of distortions. Bento and Restuccia (2017), Fattal-Jaef (2022), and Ayerst, Nguyen, and Restuccia (2024) document that more productive firms systematically face higher distortions, implying resources are reallocated toward less productive firms. While these studies emphasize the consequences for innovation and firm size distributions, we extend this logic to quantify the conditions under which productivity-dependent distortions overturn firm rankings. We show that maintaining productivity dependence without rank reversal yields much smaller aggregate gains from improved allocation.

This perspective also sheds light on studies evaluating the aggregate impact of specific distortions. Work on size-dependent regulations, such as Guner, Ventura, and Xu (2008), Garicano, Lelarge, and Van Reenen (2016), García-Santana and Pijoan-Mas (2014), and Bachas, Fattal Jaef, and Jensen (2019), finds relatively modest *TFP* effects. Our framework explains this by noting that these distortions compress firm size differences but rarely generate rank reversal, and therefore produce only limited misallocation costs.

Similarly, our emphasis on rank reversal clarifies the circumstances under which financial frictions lead to substantial productivity losses. Models with collateral constraints (Buera, Kaboski, and Shin (2011), Moll (2014), Midrigan and Xu (2014), D'Erasmo and Moscoso Boedo (2012)) show heterogeneous aggregate effects depending on parameterization. We demonstrate that these frictions yield large *TFP* losses precisely when they

[3] Salient work characterizing misallocation across countries includes Bartelsman, Haltiwanger, and Scarpetta (2013), Neumeyer, Sandleris, and Wright (2009), Busso, Madrigal et al. (2013), Cirera, Fattal-Jaef, and Maemir (2019), and de Nicola, Loayza, and Nguyen (2024), among many others.

induce sharp declines in the size-productivity rank correlation.

Taken together, this literature suggests that distinguishing between misallocation that merely compresses firm sizes and misallocation that overturns productivity rankings is critical for understanding aggregate outcomes. Our paper provides the first systematic cross-country quantification of this distinction.

3. Model

The model closely follows Hsieh and Klenow (2009), featuring multiple sectors, each populated by a fixed mass of heterogeneously productive establishments operating under monopolistic competition. Households consume a final good that is a Cobb-Douglas aggregate of sectoral goods, and each sectoral good is composed of differentiated varieties produced by firms.

Preferences are represented by a CES-Cobb-Douglas utility function:

$$U(C) = \prod_{s=1}^{S} Y_s^{\theta_s} \tag{1}$$

where $\sum_s \theta_s = 1$, and Y_s denotes the composite output of sector s. The final good C is assembled from sectoral goods with expenditure shares θ_s.

Within each sector, output is a CES aggregator over M_s differentiated firm-level varieties:

$$Y_s = \left[\sum_{i=1}^{M_s} y_{si}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

where $\sigma > 1$ is the elasticity of substitution across varieties.

Each firm i in sector s produces its variety using capital and labor inputs in a Cobb-Douglas production function:

$$Y_{si} = A_{si} L_{si}^{1-\alpha_s} K_{si}^{\alpha_s} \tag{3}$$

where A_{si} denotes firm-level physical productivity (TFPQ), K_{si} is capital input, L_{si} is labor input, and α_s is the capital share in sector s.

Capital and labor inputs are chosen to maximize profits every period, taking the capital rental rate and the wage rate in factor markets as given. While we assume no adjustment costs, we introduce wedges that distort the aggregate scale of the firm and the relative price between capital and labor. These are the output wedge $\tau_{y_{si}}$ and the capital wedge $\tau_{k_{si}}$. Importantly, the wedges are assumed to be idiosyncratic to the firm, capturing the idea that the frictions and policies may exert a heterogeneous impact on the firms' input choices. Given the monopolistically competitive behavior of the variety producers, each firm maximizes

$$\pi_{si} = (1-\tau_{Ysi})P_{si}Y_{si} - wL_{si} - (1+\tau_{Ksi})RK_{si} \tag{4}$$

$$s.t.$$

$$P_{si} = Y_{si}^{-\frac{1}{\sigma}} P_s Y_s^{\frac{1}{\sigma}} \tag{5}$$

where equation 5 is the demand for variety i in sector s and the firm's output is given by equation 3. Solving the optimization problem yields:

$$L_{si} \propto \frac{A_{si}^{\sigma-1}(1-\tau_{Y_{si}})^{\sigma}}{(1+\tau_{K_{si}})^{\alpha_s(\sigma-1)}} \tag{6}$$

$$\frac{K_{si}}{L_{si}} = \frac{\alpha_s}{(1-\alpha_s)} \frac{w}{R} \frac{1}{(1+\tau_{k_{si}})} \tag{7}$$

Equations 6 and 7 show the direction in which the wedges distort the decisions away from the efficient level. Under no distortions, firm size is determined by the firm's *TFPQ*, A_{si}, and capital-labor ratios are equalized within industries. With distortions, both properties break down, leading to resource misallocation.

The optimality conditions in the model imply that the revenue productivity of the firm, $TFPR_{si}$, represents a summary statistic of the mix of capital and output wedges. Through a simple rearrangement of terms, it can be shown that revenue productivity, defined as $TFPR_{si} = \frac{P_{si}Y_{si}}{L_{si}^{1-\alpha_s}K_{si}^{\alpha_s}}$, becomes proportional to the ratio of distortions in the following fashion:

$$TFPR_{si} \propto \frac{(1+\tau_{k_{si}})_s^{\alpha}}{(1-\tau_{Y_{si}})} \tag{8}$$

This representation of *TFPR* proves to be very useful for characterizing misallocation in an economy. Since, in the efficient allocation with no distortions, the *TFPR* must be equalized across firms, any dispersion in revenue productivity is a sign of misallocation. Furthermore, the level of a given firm's *TFPR* reveals information on the direction in which the distortions are affecting the firm relative to the average in its industry. A high *TFPR* is indicative of an inefficiently low level of labor and capital flowing to the firm, whereas the opposite is true if *TFPR* is lower than the average.

The empirical analysis in the following sections requires estimates of the firms' physical productivity, A_{si}. However, the databases at our disposal do not provide firm-level data on prices, precluding a direct inference of the firms' productivity. To circumvent this limitation, we follow Hsieh and Klenow (2009) again and estimate productivity based on data on value added, capital, labor, and model-implied prices derived from profit maximization. Such an approach yields:

$$A_{si} \propto \frac{(P_{si}Y_{si})^{\frac{\sigma}{\sigma-1}}}{(L)_{si}^{1-\alpha_s}K_{si}^{\alpha_s}} \tag{9}$$

A limitation of this approach is that any deviation from CES pricing in the data would be confounded with policies and distortions in the identification of wedges.

Aggregating labor, capital, and output across firms, taking as given the stock of labor and capital allocated to each sector s, renders the following expression for Total Factor Productivity:

$$TFP_s = \frac{\left[\Sigma_{i=1}^{M_s}\left(A_{si}\frac{\left(1-\tau_{Y_{si}}\right)}{\left(1+\tau_{K_{si}}\right)^{\alpha_s}}\right)^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}}{\left[\Sigma_{i=1}^{M_s}A_{si}^{\sigma-1}\frac{\left(1-\tau_{Y_{si}}\right)^{\sigma}}{\left(1+\tau_{K_{si}}\right)^{\alpha_s(\sigma-1)+1}}\right]^{\alpha_s}\left[\Sigma_{i=1}^{M_s}A_{si}^{\sigma-1}\frac{\left(1-\tau_{Y_{si}}\right)^{\sigma}}{\left(1+\tau_{K_{si}}\right)^{\alpha_s(\sigma-1)}}\right]^{1-\alpha_s}} \tag{10}$$

3.1. Defining Rank Reversal

As derived from the model (equations 6 and 7), firm employment is proportional to physical productivity (A_{si}) and a combination of output and capital distortions. In an undistorted economy, the ranking of firm sizes aligns perfectly with the ranking of productivity. Distortions can disrupt this alignment, potentially leading to rank reversal, as formalized below.

DEFINITION 1. *Consider two firms i and j in the same 4-digit sector, with productivities A_{si} and A_{sj} and employment L_{si} and L_{sj}. A* rank reversal *occurs if the more productive firm is smaller:*

$$\log A_{si} > \log A_{sj} \quad \textit{but} \quad \log L_{si} < \log L_{sj}.$$

This definition delivers necessary and sufficient conditions for distortions to induce rank reversal, as established by the following lemma.

LEMMA 1. *Let $A_{sj} > A_{si}$ for any pair of firms i and j in industry s. Then*

$$\frac{L_{sj}}{L_{si}} < 1 \Longleftrightarrow \frac{(1-\tau_{y_{si}})^{\sigma}(1+\tau_{k_{sj}})^{\alpha_s(\sigma-1)}}{(1-\tau_{y_{sj}})^{\sigma}(1+\tau_{k_{si}})^{\alpha_s(\sigma-1)}} > \left(\frac{A_{sj}}{A_{si}}\right)^{\sigma-1}.$$

COROLLARY 1. *If distortions operate solely through output wedges, then*

$$\frac{L_{sj}}{L_{si}} < 1 \iff \frac{TFPR_{sj}}{TFPR_{si}} > \left(\frac{A_{sj}}{A_{si}}\right)^{\sigma-1}.$$

PROOF. The lemma follows directly from the optimal employment condition in equation 6 and rearranging. The corollary follows from noting that when capital wedges are set to zero, $TFPR_{si} \propto \frac{1}{(1-\tau_{ysi})}$ □

Lemma 1 shows that for the size ranking of a more productive firm to be reversed relative to a less productive one, the former must face sufficiently larger output and capital distortions to offset its productivity advantage. Corollary 1 simplifies this condition to a direct comparison between relative *TFPR* and relative *TFPQ*.

3.2. Productivity-Dependent Distortions and Rank Reversal

A common feature of misallocation across countries is that resources tend to flow from more productive to less productive firms.[4] Such distortions are often described as "size-dependent," "correlated," or "productivity-dependent," reflecting their systematic relationship with a firm's underlying productivity or efficient scale.

Productivity dependence plays a central role in rank reversal. Lemma 1 shows that a sufficiently higher distortion on a more productive firm relative to a less productive one is required to overturn the efficient size ranking. This dependence may be local, affecting only specific firm pairs, or systematic, pervading the entire economy. Stronger and more pervasive productivity dependence amplifies the rank-reversing effects of idiosyncratic distortions.

To characterize the extent of productivity dependence, we parameterize distortions as follows[5]:

$$TFPR_{si} = A_{si}^{\eta}\varepsilon_i \tag{11}$$

Equation (11) decomposes distortions into two components. The first captures the systematic dependence on productivity, governed by the elasticity parameter η. The second is an idiosyncratic random component, ε_i, drawn from a known i.i.d. distribution $\Gamma(\varepsilon_i)$.

Rank reversal can emerge either from systematic productivity dependence or from realizations of the random component of distortions. When the slope of the distortion profile with respect to productivity, η, is sufficiently steep, systematic rank reversal occurs: the relationship between productivity and size is overturned. This extreme form of

[4] See, for instance, Bento and Restuccia (2017), Fattal-Jaef (2022), Ayerst, Nguyen, and Restuccia (2024).

[5] This parameterization of distortions follows Bento and Restuccia (2017), Buera and Jaef (2025), and Ayerst, Nguyen, and Restuccia (2024)

dependence not only generates pervasive rank reversal but can also radically alter firm dynamics. For instance, in an extension of the Hsieh–Klenow framework with fixed costs and endogenous exit, the usual prediction that the least productive firms exit flips—profits may decline with productivity, forcing the most productive firms out of the market. By contrast, even if distortions are independent of productivity on average, local rank reversal can still occur through particular realizations of the random component.

Having established the theoretical conditions for rank reversal, we now turn to the data to evaluate its prevalence and to distinguish the roles of systematic productivity dependence from random distortions.

4. Data

This section describes the firm-level datasets used to infer idiosyncratic distortions and study rank reversal in the manufacturing sector, where comprehensive firm-level data are most readily available. To examine misallocation across diverse economies, we combine manufacturing censuses and industrial surveys from national statistical agencies with firm-level financial and operational data from Amadeus (Bureau van Dijk), covering public and private firms across 43 European countries. For each country, we use cross-sectional data from a single year, ranging from 2003 for Ghana to 2014 for multiple countries, reflecting data availability from national sources.

Our sample includes the following countries, grouped by data source. For Ghana, Ethiopia, Chile, Colombia, Peru, India, Bangladesh, Pakistan, and El Salvador, we rely on national manufacturing surveys or censuses. For Belgium, Bulgaria, Finland, France, Hungary, Italy, Portugal, Romania, and Spain, we extract firm-level data from Amadeus.

We select a subset of 9 European countries from the broader Amadeus universe based on the representativeness of their firm size distribution. Following the methodology in Kalemli-Özcan et al. (2024), we compare employment shares across size categories—small, medium, and large—between Amadeus and Eurostat's Structural Business Statistics. We retain countries for which Amadeus closely replicates the official size distribution.

To harmonize across sources, we restrict our sample to firms with at least 10 employees, following the approach in Fattal-Jaef (2022). While Amadeus does not apply an explicit employment threshold, most industrial surveys and censuses include only firms exceeding a size cutoff, typically set at 10 workers in the countries we study. We apply this threshold uniformly across all datasets to ensure consistency in coverage. The list of countries, data sources, and corresponding years is reported in table A1 in appendix A.

5. Quantitative Analysis

The structure of the model follows Hsieh and Klenow (2009), enabling a straightforward calibration strategy. We set the elasticity of substitution across varieties, σ, to 3, consistent with standard estimates in the literature. The sector-specific labor share, $1-\alpha_s$, is calibrated using the value-added share of labor in U.S. manufacturing, with details on ISIC industry computations in Appendix A.1. The Cobb-Douglas utility weights θ_s in equation 1 are set to the share of value added in each industry relative to total manufacturing value added, computed separately for each country.

5.1. Characterization of Rank Reversal

To quantify rank reversal, we compare rank correlations between firm size (L_{si}) and productivity (A_{si}) in the observed allocation versus a counterfactual efficient allocation, where size rankings perfectly align with productivity rankings. In a distorted economy, correlations may fall below one depending on the properties of the distribution of distortions. When distortions fail to satisfy the condition in Lemma 1, rankings remain perfectly aligned. Only when rank reversal is prevalent does the correlation deviate from one. Accordingly, we measure rank reversal by the following correlation coefficient:

$$Corr\left(Rank_{A_{si}}, Rank_{L_{si}}\right)$$

Figure 1 shows rank correlations across countries against aggregate TFP gains from removing misallocation. Correlations range between 0.25 and 0.75, indicating substantial rank reversal. As expected, lower correlations are associated with larger efficiency gains, highlighting the central role of rank reversal in driving aggregate productivity losses.

To isolate the quantitative role of rank reversal in the aggregate losses from misallocation, we appeal to the proposed decomposition of *TFPR* into a systematic dependence on *TFPQ* and a random component, as described in equation 11. A first step in this direction is to estimate the productivity-elasticity of distortions, η, from the data. Taking logs, we get the following regression equation:

$$log\left(\frac{TFPR_{si}}{\overline{TFPR_s}}\right) = \eta * log\left(\frac{A_{si}}{\overline{A_s}}\right) + \nu_i \tag{12}$$

where the demeaned values of *TFPR* and *TFPQ* can be computed from the data based on equations 8 and 9. Figure 2 reports the estimated elasticity against GDP per capita.

The estimates confirm earlier findings of a strong negative relationship between the productivity elasticity of idiosyncratic distortions and economic development. However, the elasticity values (0.16-0.55) fall below the threshold required to generate rank reversal.

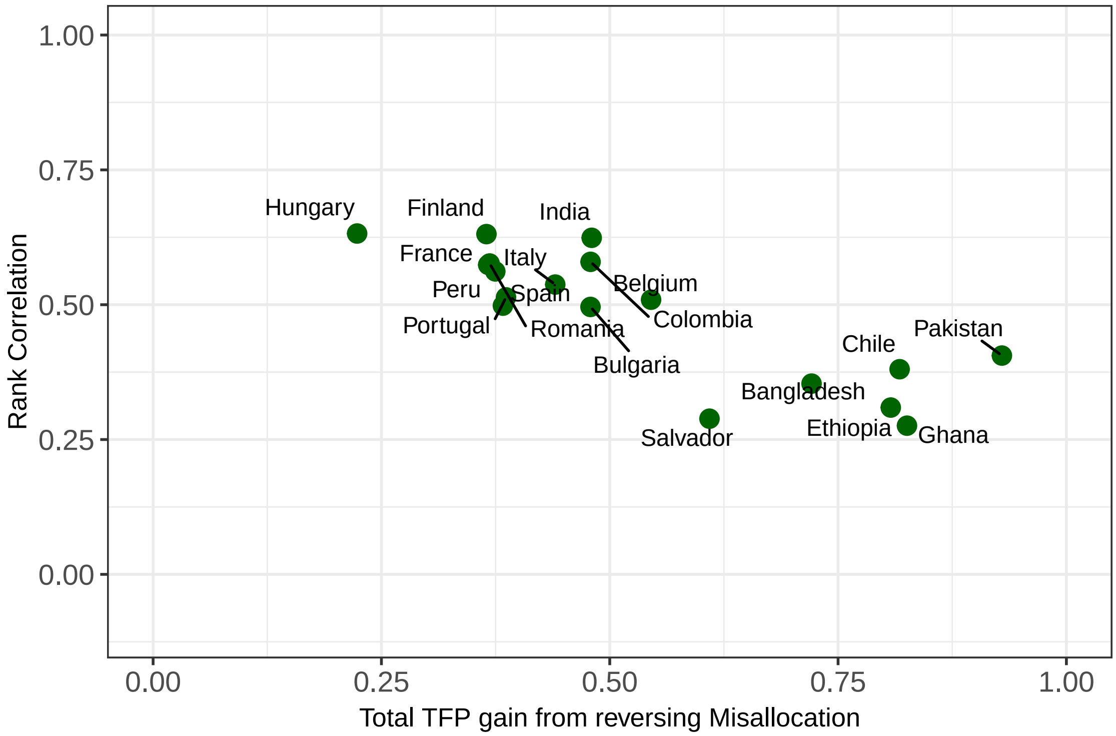

FIGURE 1. Rank Reversal and TFP Gains from Resolving Misallocation

The figure plots the correlation between firm size and productivity rankings (vertical axis) against TFP gains from removing distortions (horizontal axis) across 17 countries. Rank correlations are computed within each 4-digit ISIC industry and averaged across industries.

In other words, the systematic relationship between *TFPR* and *TFPQ* alone cannot explain the reversion of size-productivity rankings in the data, suggesting an important role for the random component of distortions.

Notice that figure 2 reports the elasticity between *TFPR* and *TFPQ*, where *TFPR* combines a mix of output and capital distortions, as shown in the equation. 8. To isolate the contribution of each specific distortion to the overall productivity dependence of *TFPR*, we can the productivity elasticity of each distortion alone, estimating the following regressions

$$log\left[\frac{\left(1-\tau_{y_{si}}\right)}{\left(1-\tau_{y_s}\right)}\right] = \eta_{\tau_y}\left(\frac{A_{si}}{\overline{A}_s}\right) + \nu_i \tag{13}$$

$$log\left[\frac{\left(1+\tau_{k_{si}}\right)}{\left(1+\tau_{k_s}\right)}\right] = \eta_{\tau_k}\left(\frac{A_{si}}{\overline{A}_s}\right) + \nu_i \tag{14}$$

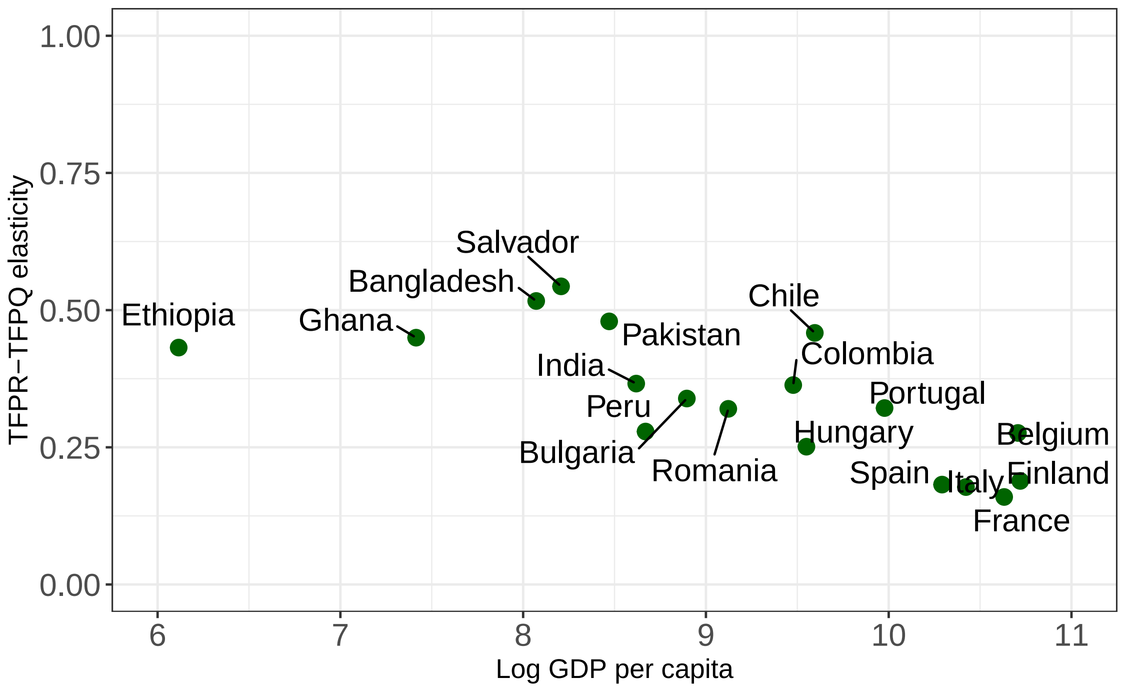

FIGURE 2. Productivity-Elasticity of Distortions and Economic Development

The figure plots the estimate of the regression coefficient between the log of demeaned values of idiosyncratic distortions and the log of demeaned idiosyncratic productivity, as specified in equation 12. These estimates are reported in the vertical axis against the log of GDP per capita in 2014 on the horizontal axis.

Equation 13 estimates the productivity elasticity of the output distortion, demeaned by its respective country-specific industrial average, whereas equation 14 estimates the same coefficient for the capital distortion, also demeaned by its country-specific industry average. From equation 8, *TFPR* is decreasing in $(1 - \tau_{ysi})$ and increasing in $(1 + \tau_{ksi})$. Therefore, abstracting from capital distortions, we should expect a negative sign for the productivity elasticity of the output distortion, to rationalize the positive elasticity between *TFPR* and *TFPQ* documented in figure 2. Similarly, abstracting from the output distortion, we should expect a positive sign for the productivity elasticity of capital distortion.

The estimated coefficients for the productivity elasticity of output and capital distortions across countries are reported in Figure 3. The left panel of the figure shows that, as expected, there is a negative relationship between the output distortion and idiosyncratic productivity. This negative elasticity implies misallocation from more to less productive firms, consistent with the direction of misallocation implied by the *TFPR-TFPQ* elasticity.

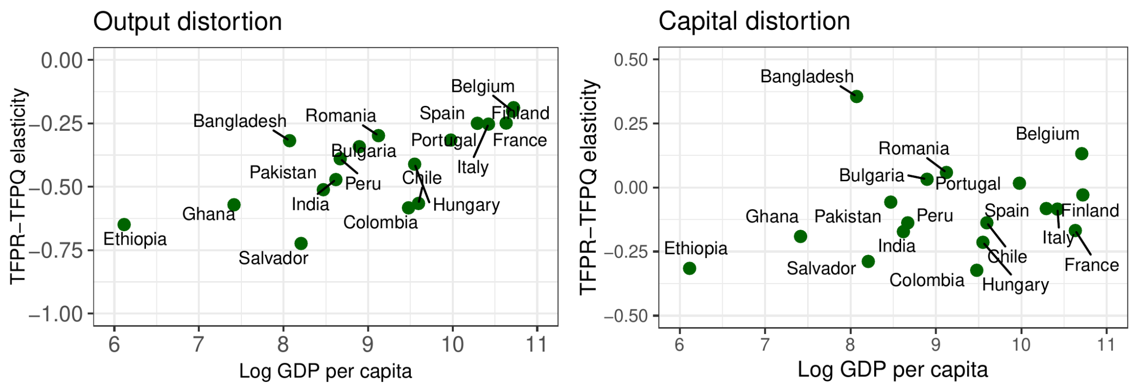

FIGURE 3. Productivity-elasticity of output and capital distortions.

The left panel plots the elasticity η_{τ_y} of output distortions from equation 13, and the right panel plots η_{τ_k} of capital distortions from equation 14, both against log GDP per capita in 2014 for 17 countries.

Moreover, the relationship between the output distortion and firm-level productivity becomes weaker in absolute value in more advanced economies, consistent with a more efficient allocation of resources in countries with higher output per capita.

The capital distortion reported on the right panel of the figure, on the other hand, exhibits a negative elasticity with respect to idiosyncratic productivity and is almost uncorrelated with economic development. Based on the capital distortion alone, labor and capital would be excessively reallocated from low- to high-productivity firms, contradicting the overall relationship between productivity and size implied by *TFPR*. Altogether, then, the strong negative relationship between the output distortion and idiosyncratic productivity dominates the weaker one between the capital distortion and productivity, rationalizing the positive relationship between *TFPR* and *TFPQ* reported in figure 2

5.2. The Quantitative Role of Rank Reversal in the Productivity Gains from Reversing Misallocation

We now turn to quantifying the role of rank reversal in shaping the aggregate productivity gains from eliminating resource misallocation. To do so, we rely on the decomposition of *TFPR* into a systematic component, reflecting productivity dependence, and a random component. The previous section showed that the average productivity dependence observed in the data is not steep enough to overturn the productivity-size ordering of firms. Rank reversal, therefore, arises only through particular realizations of the random component. This observation provides a basis for isolating the contribution of rank reversal to misallocation.

Our strategy is to construct counterfactual distributions of output and capital distortions that preserve the estimated productivity dependence but abstract from the random component. Concretely, we generate distortions predicted solely by their productivity elasticities, while eliminating the idiosyncratic realizations that generate rank reversal. This delivers a distorted allocation that mirrors the observed productivity dependence but rules out rank reversal, allowing us to assess quantitatively how much the aggregate gains from reversing misallocation depend on the presence of rank reversal.

A limitation of this strategy is that shutting down the random component is a stronger assumption than just muting the rank reversal. In principle, one could construct productivity-dependent distortions that generate more dispersed marginal revenue products of labor and capital, and still avoid rank reversal, leading to higher efficiency gains when removed. Our approach, while offering a more tractable implementation, should be interpreted as providing an upper bound on how much lower the *TFP* gains from removing misallocation can be in the absence of rank reversal.

The counterfactual output and capital distortions free of rank reversal are constructed as predicted values from the regressions in equations 13 and 14. We denote these counterfactual distortions as $\widehat{\left(1-\tau_{y_{si}}\right)}$ and $\widehat{\left(1+\tau_{k_{si}}\right)}$ respectively. Plugging these predicted values into expression 8, we derive the counterfactual distribution of Total Factor Revenue Productivity as:

$$\widehat{TFPR}_{si} \propto \frac{\widehat{\left(1+\tau_{k_{si}}\right)}^{\alpha_s}}{\widehat{\left(1-\tau_{y_{si}}\right)}} \tag{15}$$

where $\propto$ indicates that aggregate sectoral constants ensuring factor market clearing are omitted for clarity.[6] This counterfactual allocation will serve as the benchmark to quantify the specific role of rank reversal in the aggregate productivity costs of misallocation.

Figure 4 compares the aggregate *TFP* gains from eliminating actual distortions (vertical axis) with those from eliminating counterfactual distortions that preserve the average productivity dependence but rule out rank reversal (horizontal axis). The figure highlights that rank reversal is a central driver of misallocation costs: average *TFP* gains decline sharply from approximately 48% in the data to only 8.5% in the counterfactual. In some countries, the productivity dependence is so severe that the productivity gains from achieving efficiency are large despite the absence of rank reversal. In Pakistan, Ghana, Chile, and Ethiopia, the *TFP* gains remain above 25%, suggesting that the policies and frictions that misallocate resources from more to less productive firms carry a significant cost for the economy.

As mentioned earlier, the specifications of distortions without rank reversal only

[6] Recall that aggregate *TFP* is determined as demeaned values of $TFPR_{si}$, thus their determination being irrelevant for the characterization of productivity gains.

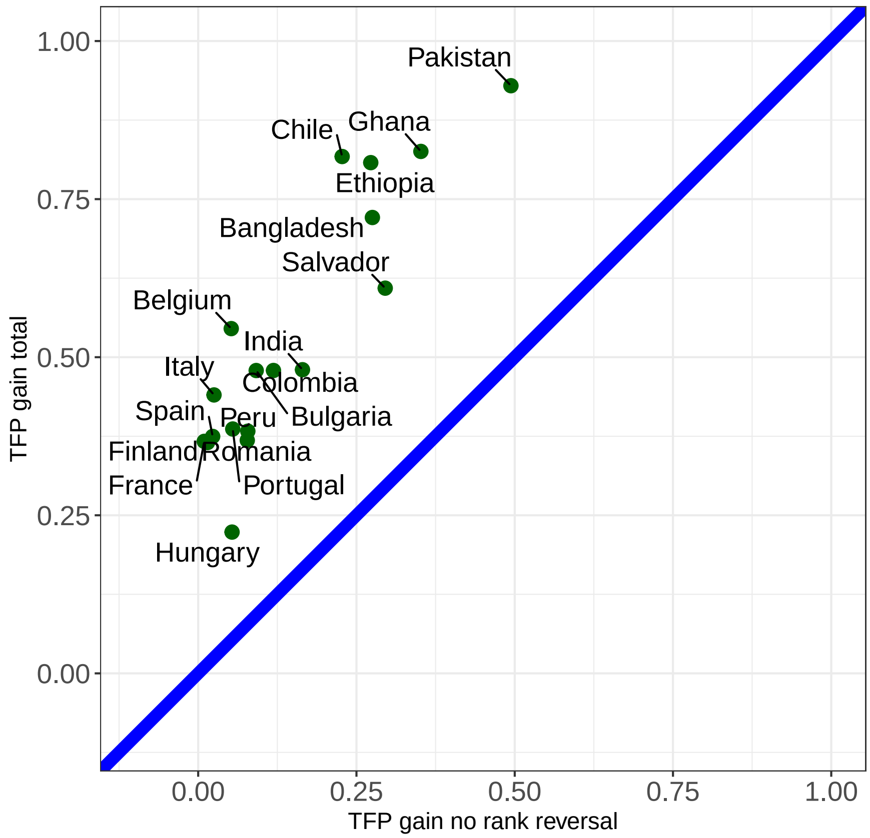

FIGURE 4. TFP Gains from Resolving Misallocation with and without Rank Reversal

The figure shows the TFP gains from removing distortions in the data (vertical axis) against the gains from removing distortions exhibiting the average productivity-dependence as in the data but without inducing rank reversal (horizontal axis

provide an upper bound for the *TFP* gap relative to the observed ones. However, the six-fold contraction in the aggregate efficiency gains when distortions are not rank-reversing merits a deeper inquiry into the drivers of rank reversal. In the following section, we discuss two candidates: the prevalence of State Owned Enterprises and financial frictions.

5.3. Drivers of Rank Reversal

We now discuss examples of policies and distortions that could, in principle, rationalize the observed rank reversal in the data. In general, most size-dependent policies examined in the literature generate productivity-dependent distortions but stop short of producing rank reversal. For instance, Guner, Ventura, and Xu (2008) studies a size-dependent tax scheme in which firms above a given employment or capital threshold face higher tax rates. In their calibrated model, resources are misallocated from more to less productive firms, yet the productivity-size ranking is preserved.[7]

Similarly, García-Santana and Pijoan-Mas (2014) analyzes the Small Scale Reservation Laws that heavily constrained factor allocation in Indian manufacturing between 1974 and 2002. These policies imposed strict caps on firm-level capital in certain industries, limiting the expansion of productive firms, but without overturning the productivity-size ordering. Finally, Bachas, Fattal Jaef, and Jensen (2019) estimates substantial size dependence in the probability of tax enforcement across firms in low- and middle-income countries. Yet even this channel, while distortionary, does not generate rank reversal. In all these cases, the associated aggregate *TFP* losses are moderate—consistent with our framework, which highlights that only rank-reversing distortions translate into the very large efficiency gains from reallocation observed in the data.

Here, we evaluate two types of frictions that have the potential of reversing size-productivity rankings: State Owned Enterprises (SOEs) and financial frictions. The existence of SOEs does not imply rank reversal by construction. However, these are often associated with excessive size and relatively lower productivity. Financial frictions, in turn, also do not necessarily generate rank reversal; however, it's a likely outcome under plausible degrees of finance-induced misallocation in low-income countries. Below, we study these distortions in greater detail.

5.3.1. Rank Reversal in State-Owned Enterprises

A subset of the countries in our firm-level dataset allows us to characterize firms by ownership type. We focus here on India's manufacturing sector in 2014, which contains the largest number of state-owned enterprises (SOEs) among all countries with ownership information. We provide complementary evidence for other countries in the appendix.

According to the 2014 Annual Survey of Industries, there are 232 SOEs in Indian manufacturing. These are defined as enterprises under (i) wholly central government ownership, (ii) wholly state and/or local government ownership, or (iii) joint ownership by central and state/local governments.

To assess the contribution of SOEs to rank reversal, we compare the distribution of rank differences between observed and efficient firm sizes for the SOE subsample against the full set of firms. Rank reversal is captured by the distribution of rank differences, where positive and negative deviations offset one another, resulting in a sum of zero in aggregate. Comparing the shape and dispersion of these distributions across SOEs and the full sample offers insight into the extent to which SOEs contribute to the observed rank reversal in Indian manufacturing.

[7] The capital–labor ratio becomes decreasing for firms whose productivity relative to the tax threshold makes it optimal to fix capital at the cutoff. However, both capital and labor allocations remain weakly increasing in productivity, and thus not rank-reverting.

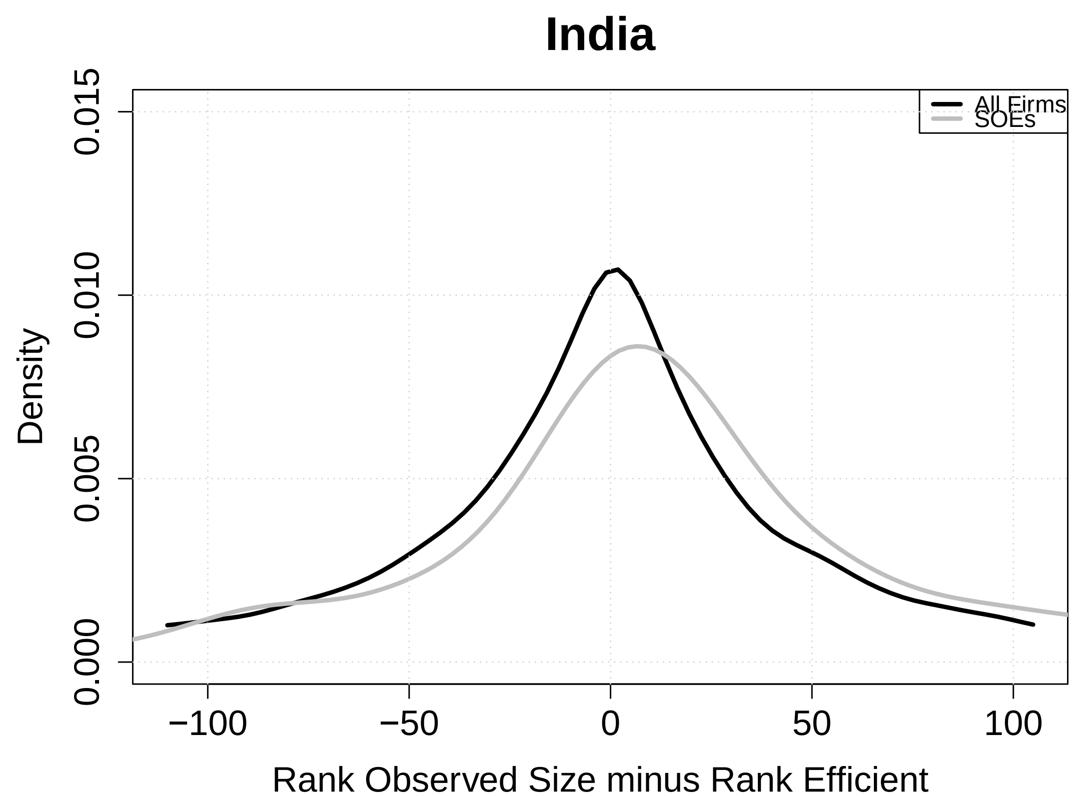

FIGURE 5. Distribution of Rank Differences in India: Full Sample versus State-Owned Enterprises.

The figure shows density estimates of the distribution of rank differences between the observed and the efficient employment sizes in India, both for the full sample of manufacturing firms in the Annual Survey of Industries for 2014, and the sub-sample of SOEs. The latter are identified as those that satisfy: (i) wholly central government ownership, (ii) wholly state and/or local government ownership, or (iii) joint ownership by central and state/local governments.

Figure 5 shows substantial dispersion of rank differences both in the entire sample and among the group of state-owned enterprises. Notably, the distribution of rank differences among the latter is shifted to the right, suggesting that firm size is predominantly larger than the efficient one compared with the overall distribution.

Despite the prevalence of rank reversal among SOEs, there is still substantial rank reversal remaining among the private firms in the economy. The correlation between observed and efficient size rankings, our headline statistic of rank reversal, is barely affected by removing the SOEs from the sample, dropping from 0.6245 to 0.624. Therefore, we conclude that state-ownership rationalizes some of the sharp misallocation from more to less productive firms observed in the data, but there's more to be explained through other frictions.[8]

5.3.2. Financial Frictions

We now turn to another prominent candidate for explaining the pervasiveness of rank reversal in the data: financial frictions. Distortions in credit markets, often in the form of collateral constraints that limit firms' access to external finance despite high marginal returns, represent a major obstacle to allocative efficiency. A large body of literature has quantified the aggregate productivity losses resulting from such frictions, including Buera and Shin (2013), Midrigan and Xu (2014), and Moll (2014). A common conclusion is that financial frictions can generate either modest or substantial misallocation, depending on the primitives that govern firms' ability to accumulate capital and escape their borrowing constraints.

We revisit this channel through the lens of rank reversal. Whereas most studies evaluate financial frictions by the dispersion of wedges or marginal products, our focus is on whether such frictions can overturn the productivity-size ranking of firms. Specifically, we ask: how much rank reversal arises under varying degrees of collateral constraints, and to what extent do the associated productivity losses map into the underlying degree of rank reversal?

To address this question, we adopt the model of financial frictions with collateral constraints from Buera and Shin (2013). In this framework, a continuum of agents draws entrepreneurial productivity from a known talent distribution, chooses between working and entrepreneurship, and saves in a risk-free asset to self-insure against idiosyncratic productivity shocks. Entrepreneurs face a collateral constraint: they can borrow up to a multiple λ of their financial wealth to finance the rental of capital stock. Tighter constraints correspond to lower values of λ, with $\lambda = 1$ implying no external financing and $\lambda \to \infty$ approaching perfect credit markets. We provide a detailed description of the model in the appendix.

An important parameter in the model is given by the stochastic process for idiosyncratic productivity. We follow Buera and Shin (2013) in assuming there's a probability ψ that the agent will preserve her entrepreneurial ability into the next period, and a probability $1 - \psi$ that she will draw a new one from the distribution. The persistent parameter is calibrated to match the exit rate across firms in the US.

Figure 6 shows that sufficiently tight collateral constraints can generate noticeable rank reversal, which in turn is associated with sizable losses in *TFP*. For example, under the tightest parameterization ($\lambda = 1.25$), the rank correlation between firm size and productivity falls to 0.85, implying a *TFP* loss of roughly 12% relative to the U.S. benchmark

[8] We reach the same conclusion when exploring rank differences distributions in other countries

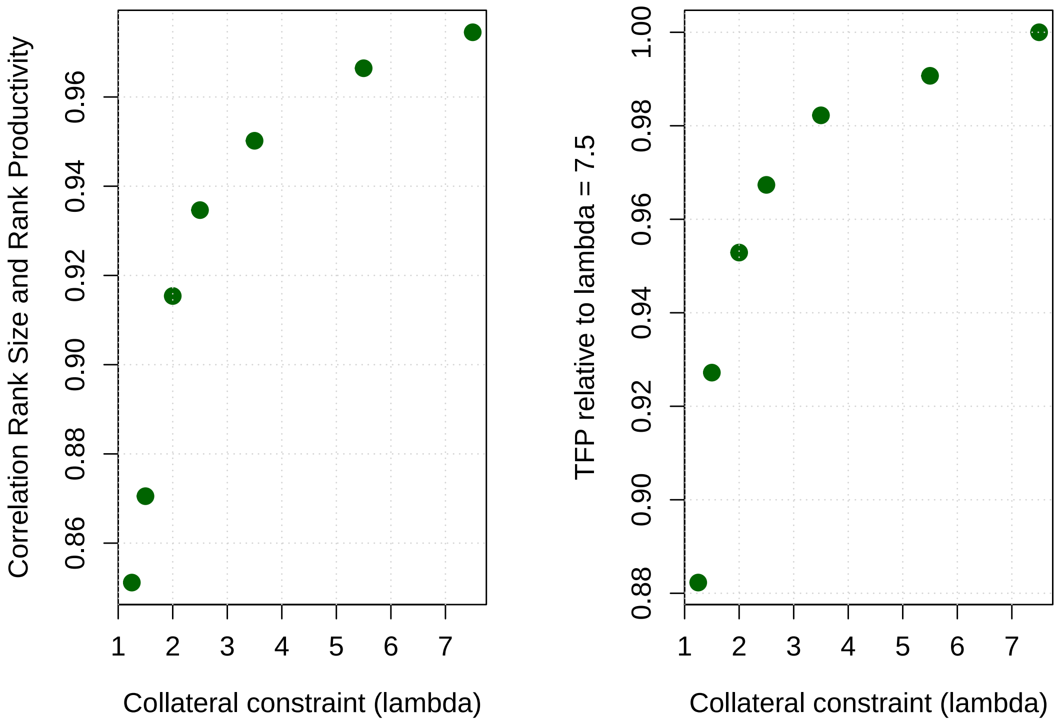

FIGURE 6. Rank Reversal and TFP Losses from Financial Frictions

The left panel illustrates the degree of correlation between actual and efficient size under various parameterizations of the collateral constraint in a model of financial frictions. The panel to the right reports the TFP losses associated with such frictions relative to an economy with collateral constraint calibrated to the US, i.e., $\lambda = 7.5$

of $\lambda = 7.5$. As constraints are relaxed, the rank correlation converges toward 1, and the associated *TFP* losses become negligible.

Two conclusions follow. First, financial frictions are capable of accounting for a meaningful share of the rank reversal observed in the data. While the median country exhibits a rank correlation of approximately 0.5, tight collateral constraints alone can reduce the correlation from 1 to 0.85, highlighting their quantitative importance, although not sufficient to fully explain the empirical patterns. Second, rank reversal proves to be an informative diagnostic of why frictions translate into modest versus large productivity losses. Financial frictions lead to substantial aggregate inefficiency only when they induce severe misalignments between firm productivity and size.

6. Conclusion

This paper has examined the properties of idiosyncratic distortions that drive aggregate productivity losses from resource misallocation. We introduced the concept of *rank reversal*—instances where firm size and productivity rankings diverge—as a diagnostic tool for assessing the severity of misallocation. We showed that economies suffering the largest *TFP* losses are also those with the most pronounced rank reversal.

The value of rank reversal emerges directly from theory. In the efficient allocation, firm size and productivity rankings coincide: competitive factor markets and costless reallocation ensure that resources flow to the most productive firms, equalizing marginal revenue products. When rankings diverge, misallocation is not just present but severe. At the same time, we showed that misallocation does not necessarily imply rank reversal. Systematic distortions may push resources away from the most productive firms without overturning relative rankings. It is the deviations from these systematic patterns that account for the rank reversal we document.

Building on this insight, we proposed a decomposition of aggregate productivity losses into those arising from systematic distortions and those attributable to rank reversal. While our measure provides only an upper bound on the latter, we find that average *TFP* gains from eliminating misallocation decrease from 50% to 14% once only the systematic component is considered. This highlights the central role of rank reversal in accounting for large efficiency losses.

We also explored specific frictions that can generate rank reversal. State-owned enterprises exhibit a significant divergence between size and productivity, yet private ownership does not eliminate this phenomenon. In addition, tight financial frictions in the spirit of Buera and Shin (2013) can generate substantial rank reversal, suggesting a promising channel for rationalizing the data.

Taken together, these findings underscore the usefulness of rank reversal as a diagnostic tool. Policies and frictions should be evaluated not only by their average impact on factor allocation but by the extent to which they sever the link between firm size and productivity. Rank reversal thus provides a clear metric for assessing the allocative cost of policy interventions.

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Appendix A. Data Appendix

TABLE A1. Firm-Level Databases: List of Countries and Data Sources

Country	Database	Description
Ethiopia	Central Statistical Agency (CSA): Large and Medium Scale Manufacturing and Electricity Industries Survey, 2011	Census of firms employing more than 10 workers
Ghana	Ghanaian Statistical Service (GSS), National Industrial Census, 2003	Census of more than 10 workers
Bulgaria, Belgium, Finland, France, Hungary, Italy, Portugal, Romania, Spain	Amadeus, 2014	(see note below for selection criteria)
Chile	ENIA (Encuesta Nacional Industrial Annual), 2013	Census of all industrial firms with 10 or more workers
El Salvador	EAM (Encuesta Anual Manufacturera), 2004	Census of all industrial firms with 10 or more workers
India	ASI (Annual Survey of Industry), 2004–2005	Census of all industrial firms with 10 or more workers in case of power usage; 20 or more workers without power usage
Colombia	EAM (Encuesta Anual Manufacturera) 2016	Census of all industrial firms with 10 or more workers
Peru	National Economic Census, 2008	Census of all firms with no size cutoff but restricts analysis to 10 workers or more
Bangladesh	Survey of Manufacturing Industries, 2012	Census of large firms, representative sample of all firms with 10 workers or more. Stratification by size class and 4-digit industry, sampling weights provided
Pakistan	Census of Manufacturing Industries, 2005	Census of registered manufacturing firms

Note: Selection criteria for the Amadeus database are as follows. Keep in sample if 1) the ratio of aggregate manufacturing employment in Amadeus and the aggregate manufacturing employment in Eurostat is greater than or equal to 80 percent, and 2) the ratio of manufacturing employment between Amadeus and Eurostat in each bin of Eurostat's size distribution above the worker threshold (10–20, 20–50, 50–250, 250+) is also greater than or equal to 80 percent. For more details see Fattal-Jaef (2022)

A.1. Sectorial Labor Shares

The sectoral labor shares are calibrated based on the US manufacturing sector. We use the NBER-CES manufacturing database Becker, Gray, and Marvakov (2021), NAICS 2012 version. To convert into the ISIC Rev.4 and the ISIC Rev. 3.1 classification systems, which are widely used in many countries, we use concordance tables from the United Nations' Statistics Division [9].

When converting from NAICS 2012 to ISIC rev4, some of the industrial codes in the former map are mapped to multiple codes in the latter. To address this, we split the value added and the payroll cost proportionally into the number of ISIC Rev. 4 codes that the corresponding NAICS 2012 is mapped into. Then, we compute the value added share of payroll payments at the four-digit ISIC Rev. 4 level.

[9] Concordance tables between NAICS2012 and ISIC Rev.4 can be downloaded at https://unstats.un.org/unsd/classifications/Econ/

Appendix B. A Model of Financial Frictions with Collateral Constraints

We characterize rank reversal in a model of financial frictions with collateral constraints in the spirit of Buera and Shin (2013). More concretely, we follow the presentation of the model in Buera, Fattal Jaef, and Shin (2015).

At the beginning of a period, an individual's state is summarized by his financial wealth a and entrepreneurial productivity z. The individual solves for its optimal savings, occupation and, if an entrepreneur, its optimal hiring of labor and rental of capital, according to the following optimization problem:

$$
\begin{aligned}
v_t(z,a) &= \max_{c,a'\geq 0} \ \xi_t \frac{c^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t v_{t+1}(z',a') \qquad \text{(A1)}\\
&\text{s.t. } \ c + a' = \max\{w_t, \pi_t(z,a;r_t,w_t)\} + (1+r_t)\,a - \tau_t
\end{aligned}
$$

where

$$
\begin{aligned}
\pi_t(z,a;r,w) &= \max_{k,l} \ zk^{\alpha} l^{\theta} - (r_t+\delta)\,k - w_t l\\
&\text{s.t.} \quad k \leq \lambda a
\end{aligned}
$$

The occupation choice of an individual is denoted by $o_t(a,z) \in \{0,1\}$. Individuals choose entrepreneurship ($o = 1$) if and only if the period's profit exceeds the hiring market wage (which is equal to the unemployment benefit): $w_t < \pi_t(z,a;r_t,w_t)$. The capital input of entrepreneurs is subject to a collateral constraint, $k \leq \lambda a$. We denote the labor and capital input choices of an entrepreneur by $l_t(a,z)$ and $k_t(a,z)$, both of which are zero for employed and unemployed workers.

The model assumes a frictional labor market where only a fraction γ of workers seeking a job will be presented with a job opportunity at the equilibrium wage rate w. However, if unemployed, the worker receives unemployment insurance equal to the equilibrium wage. For more details on the characterization of the equilibrium, see Buera, Fattal Jaef, and Shin (2015).

Calibration. We focus on the stationary equilibrium of the model. We calibrate parameters targeting the US economy. Then, we experiment with alternative values of the collateral constraints, keeping the remaining parameter values unchanged. The resulting calibration is summarized in Table A2

TABLE A2. Calibration

	US Data	Model	Parameter
Top 10% Employment Share (Firms)	0.78	0.76	$\eta = 3$
Top 5% Earnings Share (Individuals)	0.36	0.35	$\alpha + \theta = 0.75$
Firm Exit Rate (Annual)	0.08	0.08	$\psi = 0.98$
Real Interest Rate (Annual)	0.04	0.04	$\beta = 0.97$
Credit Market Instruments to Non-Financial Assets	0.59	0.57	$\lambda = 7.5$

Note: The employment share of the top 10 percent of firms and the firm-based exit rate are from the Business Dynamics Database for the year 2007 (U.S. Census Bureau 2025). The top 5 percent earnings share is from the Survey of Consumer Finance for the year 2013.

Appendix C. Rank Reversal among State-Owned Enterprises

The text characterized rank reversal among SOEs in India, where the number of such firms in the manufacturing sector is the largest across countries with ownership data in our sample. Here, we complement India's evidence with a characterization of rank reversal among SOEs in Ghana, Bangladesh, and Pakistan. In all cases, we refer to SOEs as those firms flagged as fully state-owned in the respective databases.

There were 240 SOEs in Pakistan's manufacturing sector in the year 2005, amounting to 6% of firms in the sample. There were 17 and 22 SOEs in Ghana and Bangladesh, respectively, which accounted for less than 1% of all manufacturing firms in the data.

Figure A1 validates India's finding in the main text. While there is also substantial rank reversal among SOEs in Bangladesh, Ghana, and Pakistan, it is insignificant compared to the rank reversion in the entire economy, accounting for a negligible portion of the overall pattern in the data.

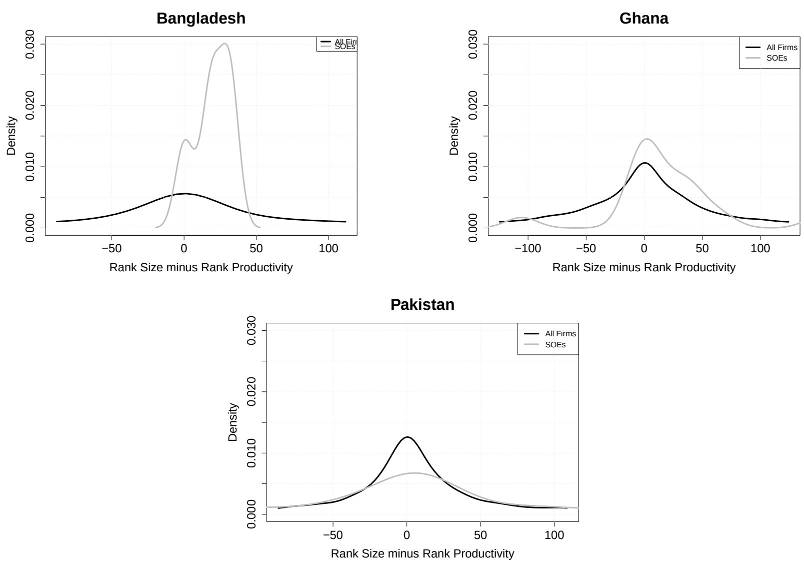

FIGURE A1. Distribution of Rank Differences: All firms vs Private Firms

The figure shows density estimates of the distribution of rank differences between the observed and the efficient employment sizes in Ghana, Pakistan, and Bangladesh, both for the full sample of manufacturing firms and the sub-sample of SOEs. The latter are identified as those flagged to be fully government-owned in the respective databases.