Distortions, Informality, and the Firm Size Distribution: Evidence from Sub-Saharan Africa

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We study firm size distributions and their implications for employment and productivity in Sub-Saharan Africa. Using micro data from Cameroon, Ghana, and Rwanda, we document that the establishment distribution is dominated by informal microenterprises, while employment is disproportionately concentrated in a few large formal firms. Conditional on formality, however, firm size and employment distributions closely resemble those in the United States. We develop a model of firm dynamics with endogenous informality that incorporates entry barriers and idiosyncratic distortions in the formal sector. The model accounts for the observed distributions and implies that the joint impact of distortions on firm growth generates aggregate *TFP* losses of up to 20 percent.

Keywords: informality, misallocation, entry barriers, size distribution, firm dynamics

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1. Introduction

One of the most important development challenges is accelerating growth in African economies to generate the jobs needed to absorb a rapidly growing population. Underlying this challenge is that a large informal sector characterizes African economies, and more importantly, African firms are smaller and create fewer formal jobs than those in other regions (see, for example, Iacovone, Ramachandran, and Schmidt (2013); Sandefur (2010)). While several factors can explain this firm dynamics, a central hypothesis is that costly business environments and distortionary policies lead to persistent inefficiencies that hinder innovation, adoption of new technologies, and firm growth.

The growing availability of firm-level datasets across countries now makes it possible to test this hypothesis by examining how such distortions shape firm dynamics. If these inefficiencies are indeed present, they should be reflected in systematic patterns at the micro level. This paper investigates this hypothesis using comprehensive firm-level data from Cameroon, Ghana, and Rwanda that includes informal sector establishments; economies where gaps in firm performance relative to advanced countries are particularly pronounced. To this end, we identify key features of firm size distributions in these countries, assess how these patterns can reveal underlying economic distortions, and quantify their broader macroeconomic implications.

We begin by documenting key differences in firm-size distributions between Sub-Saharan Africa and the United States.[1] In Cameroon, Ghana, and Rwanda, micro-enterprises are predominant, whereas medium- and large-sized firms play a much more significant role in the U.S. economy. The distribution of employment across establishments is also skewed toward smaller units, although the contrast with the United States is less stark in this case. We then show that the widespread presence of informal establishments largely accounts for these differences. When the analysis is restricted to formal producers, firm size and employment distributions more closely mirror those observed in the United States. Finally, we find that employment growth over the firm life cycle is limited, as suggested by the cross-sectional age distribution of establishments.

Next, we introduce a standard model of firm dynamics featuring endogenous determination of formal and informal sector participation, firm entry and exit, and innovation. This framework allows us to identify the distortions that may underlie the firm-size distributions documented earlier. We focus on Ghana, where detailed micro-data on firm size are available alongside information on value added, capital stocks, and labor costs. Using this rich dataset, we identify idiosyncratic distortions and barriers to formal sector entry—following the approach in Hsieh and Klenow (2009) for the former and Fattal-Jaef (2022) for the latter. Our analysis reveals significant resource misallocation and substantial

[1] As will be explained later, the empirical unit of analysis is the establishment. However, we use the terms firms and establishments interchangeably.

entry barriers. In particular, factor inputs are not efficiently allocated across firms, as indicated by the measured elasticity between idiosyncratic distortions and firm-level productivity (*TFPQ*). Moreover, our model-based estimation implies that, to replicate the observed average establishment size in equilibrium, a sizable entry tax is required.

Finally, we assess the model's ability to replicate the key features of the firm-size distribution and to quantify the macroeconomic implications of the underlying distortions. We find that the interaction of these distortions closely reproduces the observed establishment size distribution and accounts for the high share of employment in micro-enterprises. Moreover, while underestimating the share of employment at the top of the distribution, the model provides a close match to the employment-based size distribution when winsorizing the top 1% of the largest firms. [2]A key factor behind the model's fit is the endogenous expansion of the informal sector in response to distortions, which leads to 95% of firms operating informally—closely aligning with the 97% observed in the data. At the macro level, the combined impact of entry barriers and idiosyncratic distortions results in a 20% reduction in total factor productivity (TFP) relative to a frictionless benchmark.

Our characterization of the establishment size distribution is based on the universe of establishments in Cameroon, Ghana, and Rwanda. A unique feature of the data is the universal coverage of all businesses operating in a fixed location and the collection of detailed information regarding the business's registration status with government agencies and accounting practices, allowing us to identify informal producers. However, the data lacks information on the firm's balance sheet. Therefore, for the quantitative analysis of distortions, we focus on Ghana's economy, which gathers financial information for all medium and large firms and a representative sample of small and micro enterprises.

We adopt a model-based approach to identify distortions and barriers to entry, and to quantify their micro- and macroeconomic implications. The model features a formal sector in which heterogeneously productive firms produce differentiated goods and compete in monopolistically competitive markets. Firm-level productivity evolves endogenously through innovation investments, and firms exit the formal sector endogenously due to fixed operating costs. The informal sector also produces a basket of differentiated goods, but in this case, firm productivity and exit are exogenously calibrated to match observed properties of the informal firm-size distribution. An infinite pool of potential entrants chooses whether to enter the formal or informal sector. Preferences are governed by constant elasticity of substitution (CES), both across varieties within each sector and between the formal and informal consumption baskets.

We focus on two types of distortions in the formal sector that we propose as key drivers of both the emergence of the informal sector and the observed shape of firm-size

[2]The top 0.1% of the distribution comprises old firms whose physical productivity is so below their observed size that our parsimonious characterization of productivity-dependent distortions cannot rationalize

distributions. The first are idiosyncratic distortions, defined following Hsieh and Klenow (2009) as wedges in the distribution of marginal revenue products across firms within narrowly defined manufacturing industries. Our analysis centers on the elasticity between these distortions (*TFPR*) and firms' physical productivity (*TFPQ*). This elasticity captures the extent of resource misallocation from more to less productive firms, which in turn affects both the incentives for new establishments to enter the formal sector and their potential to grow. The second distortion is a barrier to entry into formal production, modeled as a tax on entry costs into the formal sector. In this case, we follow Fattal-Jaef (2022) by identifying the magnitude of the entry barrier required for the model's stationary equilibrium to replicate the average firm size observed in the data, conditional on the presence of idiosyncratic distortions.

The model with distortions can rationalize the main features of the firm size distributions in our Sub-Saharan African sample only when both distortions are at play. When entry barriers are operating alone, there's a counterfactual concentration of employment on large firms, well beyond the employment concentration at the top of the size distribution in the U.S. economy. Conversely, when there are only productivity-dependent distortions, there's an excessive concentration of employment in the middle of the size distribution, reflecting the distortions expanding the size of the less productive and shrinking the more productive producers. When both distortions interact, on the other hand, each distortion's limitation is mitigated, and the overall size distribution approaches the observed one. Still, the measured productivity dependence of idiosyncratic distortions reallocates too many resources away from large firms, creating a shortage of employment at the top. Many of the largest firms in Ghana are substantially less productive than their average industry counterparts, requiring a sizable production subsidy that the estimated slope of the *TFPR* versus *TFPQ* cannot generate.

The remainder of the paper is organized as follows. Sections 2 and 3 describe our contribution to the literature and the main features of our data. Section 4 presents the empirical regularities distinguishing the firm size distributions in Sub-Saharan Africa. Section 5 introduces the quantitative model to extract idiosyncratic distortions and entry barriers, and section 6 conducts the quantitative evaluation of the model's fit to the observed size distribution and derives the macroeconomic implications of the distortions. Section 7 concludes.

2. Contributions to the Literature

Our paper contributes to the growing literature in development macroeconomics, leveraging microdata to learn about distortions and extract macroeconomic implications. It is also closely related to the narrower branches of this agenda, which focus on informality.

There is a dense literature documenting cross-country differences in the firm size

distribution and proposing distortions to rationalize these. Alfaro, Charlton, and Kanczuk (2008) and Hsieh and Klenow (2009) are pioneers in this field. More recently, Poschke (2014), Bento and Restuccia (2017), and Fattal-Jaef (2022) have built on the earlier research by bringing new margins of adjustment to understand the effect of policies and distortions on the firm size distribution. Our paper builds upon the methodological contributions of this literature, notably Hsieh and Klenow (2009) and Fattal-Jaef (2022), to implement their strategies to identify distortions in newer contexts, in our case, Sub-Saharan Africa. A contribution to these studies is the consideration of the effect of distortions on the informal sector.

In addition, there is an important debate in the literature regarding the existence of a missing middle in the firm size distribution of less developed countries. Salient exponents in this literature are Tybout (2000), Hsieh and Olken (2014), and Tybout (2014). The existence of a missing middle in the firm size distribution has also been explored in the context of Sub-Saharan Africa by Tsaedu, Chen, and Azmete (2023) and Teal (2023). Our paper broadens the focus on firm size distributions as diagnostic tools for distortions beyond the missing middle debate and focuses on distortions affecting the entire firm distribution. We characterize the systematic differences between the firm size distributions across low and high-income countries. Adopting this broader perspective helps us highlight the key aspects of underdevelopment: the prevalence of informality and the lack of life-cycle firm growth. Moreover, rather than speculating about policies that could rationalize the patterns of size distributions, we provide an identification strategy for distortions generating these patterns and assess their merit in the context of a quantitative model.

Our paper also relates to the vast literature on the macroeconomics of informality. In particular, we relate most closely with the research postulating quantitative models of informality to evaluate the implications of specific distortions and policies. Ulyssea (2018) study the effect of taxes and regulations in a rich setting with informal firms and informal hiring within formal firms. D'Erasmo and Moscoso Boedo (2012), Antunes and Cavalcanti (2007), and Lopez-Martin (2019) study informality in the context of financial frictions, whereas Alvarez and Ruane (2019) and Dabla-Norris et al. (2018) characterize the effect of labor market regulations.

Our model extends the literature regarding the interaction of distortions and the treatment of informality. First, we postulate the interaction between productivity-dependent distortions and entry barriers, identified from the microdata, as drivers of the observed informality. Idiosyncratic distortions capture the full extent of misallocation in the economy, encompassing the role of financial frictions, labor regulation, and any other policy that manifests as a productivity-dependent idiosyncratic distortion. Moreover, we validate the model against the documented establishment and employment-based size distribution

patterns, separating implications for informal and formal firms' distributions. Finally, we innovate how we model formal sector firm dynamics and informality. Given the observation of weak firm growth over the life cycle of formal firms, we model firm dynamics as endogenously arising from firms' innovation decisions. Regarding informality, we postulate informal sector goods as a differentiated set of varieties imperfectly substitutable in consumers' preferences.

Last, our modeling approach follows the interpretation of the informal sector in La Porta and Shleifer (2014), where informal firms are considered a low-productivity group of entrepreneurs without a significant potential for growth. As in La Porta and Shleifer (2014), in our model, the informal sector expands and becomes prevalent as a result of barriers to entry and allocative distortions in the formal sector, which reduce wages, increase the relative price of formal goods, and redirect entry into the informal sector. When distortions are removed, the contraction of the informal sector emerges not as a result of the formalization of existing firms, but due to increased formal sector entry that increases labor demand for former informal entrepreneurs.

3. Data

The lack of suitable data is one of the main challenges in analyzing the firm size distribution to assess broader macroeconomic patterns, constraints to productivity growth, and the underlying market frictions and policy distortions. These challenges are most acute in Sub-Saharan Africa, a region where most of the low-income economies are and where these constraints are likely to be most pressing.

Over recent years, a number of Sub-Saharan African countries have collected establishment censuses as part of wider programs aimed at informing economic policies. These censuses are typically business registers and include contact and location information, legal status, business registration, tax and other administrative records, type of economic activity, and number of workers. Crucially for our purpose, the censuses cover all establishments, including unregistered ones, so the ensuing data is a more accurate proxy of the true firm population.[3]

This paper uses establishment censuses from three sub-Saharan African countries: Cameroon, Ghana, and Rwanda. We now turn briefly to describe each dataset.

[3] It is worth noting the limitations of the data. First, it is cross-sectional and does not allow for the disentangling of the contributions of firm growth and survival. This limitation restricts the analysis of the life-cycle dynamics of firms to be inferred from the cross-sectional age distribution, as in most of the literature. Second, it lacks rich financial information about the establishment, precluding analysis of the dynamics and drivers of productivity. Finally, the data do not allow decomposing formal and informal employment within a formal firm, an intensive margin that was highlighted as important in (Ulyssea 2020, 2018). Notice, however, that this distinction does not affect the size distribution pattern but interferes with the breakdown of such distribution into formal and informal producers.

Cameroon. The data is obtained from the first Recensement Général des Entreprises (RGE) conducted in 2009 by the Institut National de la Statistique (INS). The RGE covers all businesses of the modern sector with a physical location. It excludes non-commercial agriculture, the street trading industry (hawkers, peddlers, and others), car and bike taxis, and the callbox industries. It provides detailed information such as registration with administrative agencies, tax payment, accounting practices, age, employment size, state ownership, and foreign equity. In the census, establishments are considered formal if they prepare a Statistical and Fiscal Declaration (DSF) or if they do not prepare a DSF but keep an operating account and a partial account of balance sheets.[4]

Ghana. The data comes from the 2014 Integrated Business Establishment Survey (IBES) the Ghana Statistical Service conducted. The IBES is an economic census of all business establishments (registered and unregistered) in the non-agriculture sectors of the economy. It has detailed information about registration with administrative agencies, tax payment, accounting practices, age, employment size, state ownership, and foreign equity. The data contains records of an establishment's registration status with government agencies, and formality is defined based on registration status with the Registrar General's Department (RGD) and keeping formal accounts.[5]

Rwanda. The source of the data is the 2014 Establishment Census (EC) conducted by the National Institute of Statistics of Rwanda (NISR). The EC constitutes a virtually exhaustive business registry as it surveys every establishment that has a fixed location and operates an economic activity. The ensuing microdata is comprehensive and detailed concerning key establishment characteristics such as location, registration, legal status, age, ownership and management, industrial classification, and employment size. The census covers both formal and informal businesses. An establishment is considered formal if it is registered with the Rwanda Revenue Authority (RRA) and maintains regular operational accounts.[6]

[4]National Institute of Statistics of Cameroon (INS). General Census of Enterprises In 2009. September 2010.

[5]Ghana Statistical Service. Integrated Business Establishment Survey: Summary Report. September 2015.

[6]National Institute of Statistics of Rwanda (NISR). The Establishment Census, 2014. June 2015.

TABLE 1. Description of the establishment census data

Indicator	Coverage	Cameroon (2008)					Ghana (2013)					Rwanda (2013)				
		Total	Foreign-owned		State-owned		Total	Foreign-owned		State-owned		Total	Foreign-owned		State-owned	
		#	#	%	#	%	#	#	%	#	%	#	#	%	#	%
Establishment	All	88,144	6,385	7.24	185	0.21	638,234	8,827	1.38	25,536	4.00	154,236	10,510	0.98	1,577	1.02
	Manufacturing	10,922	519	4.75	1	0.01	99,437	711	0.72	164	0.16	10,742	152	1.42	8	0.07
Employment	All	429,758	73,464	17.09	9,590	2.23	3,383,206	136,265	4.03	497,078	14.69	493,302	28,920	5.86	67,546	13.69
	Manufacturing	82,502	20,742	25.14	352	0.43	437,316	31,272	7.15	5,657	1.29	39,708	6,883	17.33	1,667	4.20

Source: Establishment censuses obtained from the statistical agencies of the select countries; see section 3. *Note:* Establishments with missing employment are excluded.

TABLE 2. Summary statistics on establishment census data

Country	**All sectors**						**Manufacturing**					
	Establishment	Employment	Employment size (# of workers)				Establishment	Employment	Employment size (# of workers)			
	Formal	Formal	Informal		Formal		Formal	Formal	Informal		Formal	
	%	%	Mean	p50	Mean	p50	%	%	Mean	p50	Mean	p50
Cameroon (2008)	13.6	54.3	3	2	20	4	8.1	61.5	3	2	57	9
Ghana (2013)	9.5	40.1	4	2	22	6	3.0	31.4	3	2	47	7
Rwanda (2013)	6.5	41.5	2	1	20	6	3.8	37.9	2	1	37	7

Source: Establishment censuses obtained from the statistical agencies of the select countries; see section 3. *Note:* Establishments with missing employment are excluded.

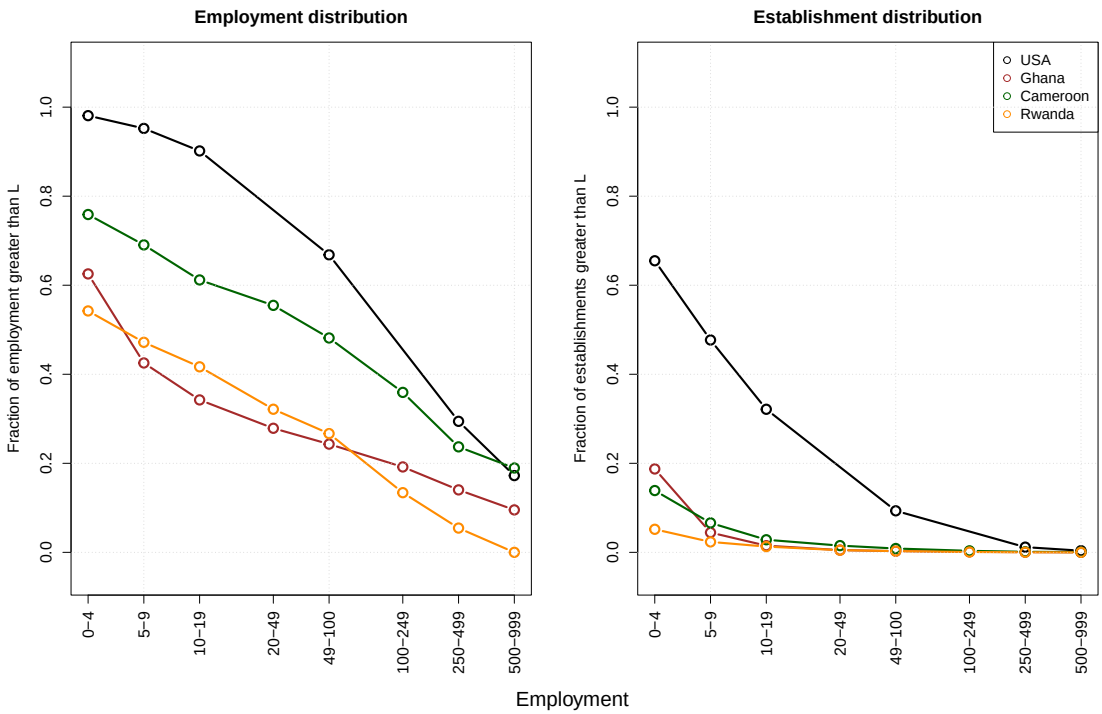

FIGURE 1. Establishment and Employment Based Size Distributions: SSA vs USA

The left panel illustrates the fraction of employment accounted for by firms larger than or equal to the size bins in the horizontal axis. The right panel illustrates the same information for the fraction of establishments.

4. Stylized Facts from Firm Size Distributions in Sub-Saharan Africa

This section highlights the salient patterns of firm size distributions in Sub-Saharan Africa. For comparison purposes, we present the results alongside the size distribution of an advanced economy such as the US. A limitation of the publicly available US data is that it presents information on the number of establishments and employment across firm-size bins rather than allowing us to work directly with the firm-level data.[7] Thus, we restrict our presentation of stylized facts in Sub-Saharan Africa to a binning strategy that mimics that in the US.

Figure 1 illustrates the main findings: the prevalence of micro-enterprises, the concentration of employment at the bottom of the size distribution, and the relative scarcity at the top. We convey these messages by plotting the right tail of the cumulative distribution functions in each country. The panel to the right shows the fraction of establishments with a labor force greater than or equal to a respective employment size category. The panel to the left depicts the fraction of employment accounted for by establishments exceeding a

[7]Our data on the firm size distribution for the US is based on the Business Dynamics Statistics for the manufacturing sector in 2022 Bureau (2022)

specific size.

The starkest contrast between Sub-Saharan African countries and the US economy relates to micro-establishment dominance. Less than 20% of establishments exceed five workers in size in Sub-Saharan Africa's manufacturing sector, significantly lower than the 60% observed in the US. This prevalence of micro enterprises is not unique to Sub-Saharan Africa but is instead a rooted feature of underdevelopment, as shown, for instance, in Hsieh and Olken (2014). As we shall see below, our contribution is to isolate the informal establishments as their driver.

The differences are less striking yet still notable in the employment-based distributions, shown in the left panel. In Ghana and Rwanda, about 60% of employment is absorbed by establishments larger than five workers, in contrast with over 90% in the US. Similarly, there's a smaller share of employment at the top of the size distribution in Ghana and Rwanda. In the US, 20% of employment is accounted for by firms larger than 1000 workers, whereas large firms in Ghana and Rwanda account for half as much. Cameroon's size distribution tracks the US one more closely, although it still features significantly more employment at the lower end of the distribution.

In Figure 2, we leverage the depth of our data to evaluate the role of informal producers in explaining the patterns of size distributions. To achieve this, we illustrate the right tails of cumulative distribution functions, considering only the formal firms in the sample. We adopt the definition of a formal establishment as provided by each country's statistical agencies, which typically define these as those registered with a government agency and adhering to formal accounting practices.[8] Table 2 shows that formal establishments represent a select group of producers in each country, amounting to only 8%, 3%, and 4% of establishments in Cameroon, Ghana, and Rwanda, respectively, and accounting for less than a third of total manufacturing employment.

The main message from Figure 2 is the closer resemblance of the size distributions of formal firms in Sub-Saharan African countries and the U.S. [9] Unlike the patterns for the whole economy, the select group of formal producers attains large scales. Perhaps surprisingly, the largest formal producers in Ghana and Cameroon attract an even larger share of the labor force than the US economy.

The similarity between the firm size distribution in Sub-Saharan Africa's (SSA) formal sector and that of the U.S. can be interpreted in three ways. First, talented formal entrepreneurs may successfully navigate the distortions that hinder formalization and growth elsewhere in the economy, effectively operating as if in an undistorted environ-

[8] Given the richness of the data, we experiment with alternative definitions of informality based on the specific combination of government agencies—such as the business registry, the tax authority, and the social security administration—with which establishments are registered. We find results similar to those derived from the data-provided flags of informality.

[9] The US's firm size distribution is assumed to consist entirely of formal establishments. Thus, its shape in the baseline figure is identical to that in Figure 2, where it is compared alongside SSA's formal sector.

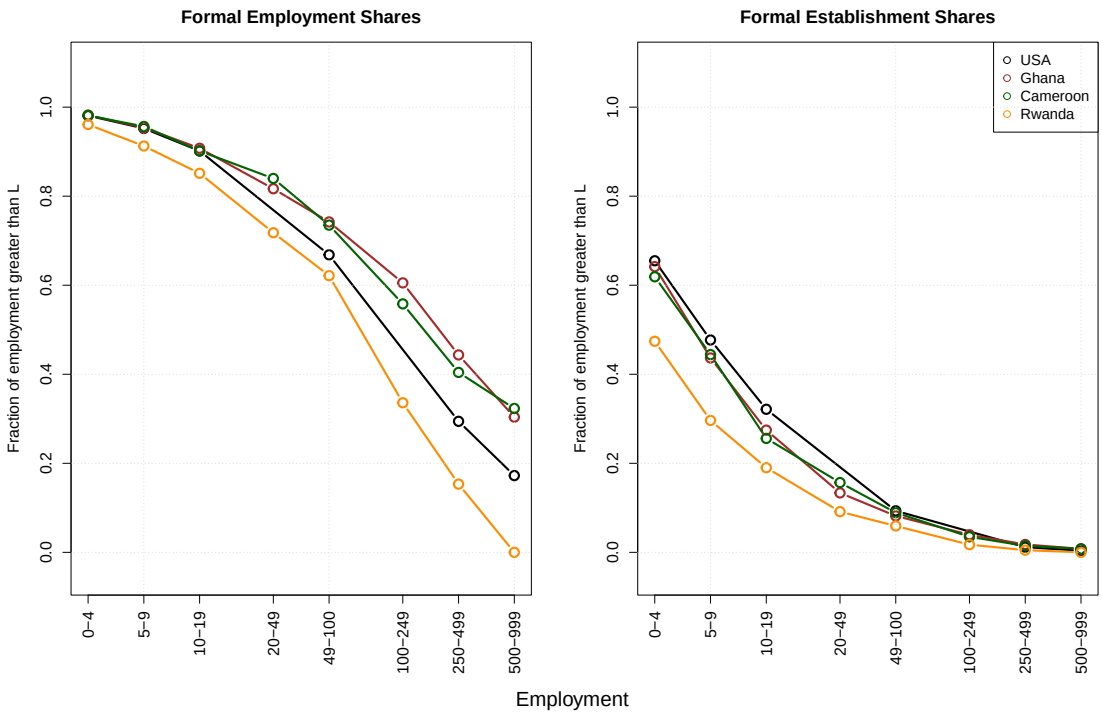

FIGURE 2. Formal Sector's Establishment and Employment-Based Size Distributions: SSA vs USA

The left panel illustrates the fraction of employment accounted for by firms larger than or equal to the size bins in the horizontal axis. The right panel illustrates the same information for the fraction of establishments. For Sub-Saharan African countries, only formal sector firms were kept in the sample

ment. Second, given the small size of the formal sector, the upper tail of the employment distribution may be disproportionately shaped by a few outliers, whose absence would significantly alter the distribution's shape. Third, contradicting the first interpretation, the observed concentration at the top may reflect a severe misallocation, where firms of lower capability thrive due to policy distortions. Using a model applied to Ghana's economy, we later demonstrate that the formal sector faces significant entry barriers and resource misallocation, lending stronger support to the third explanation over the first.

Figure 3 explores the influence of outliers by presenting firm and employment size distributions for the formal sector using a winsorized dataset.[10] The figure confirms that outliers do shape the formal sector's distribution, yet the overall resemblance to the U.S. distribution remains evident.

We conclude the empirical section of the paper by examining differences between the size distribution of young establishments—those less than five years old—and that of the entire economy. This analysis aims to assess the extent of business dynamism through the

[10]After isolating formal firms from the original database, we winsorize the top portion of the establishment size distribution.

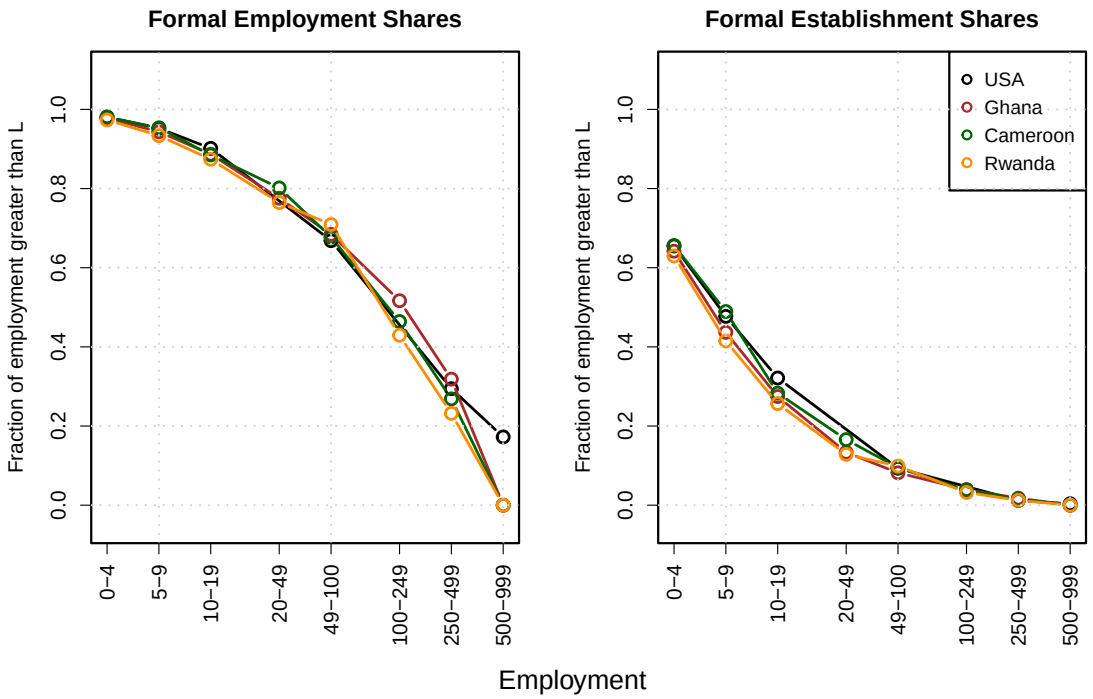

FIGURE 3. Formal Sector's Establishment and Employment-Based Size Distributions under Winsorized Data: SSA vs USA

The left panel illustrates the fraction of employment accounted for by firms larger than or equal to the size bins in the horizontal axis. The right panel illustrates the same information for the fraction of establishments. Only formal sector firms were kept in the sample for Sub-Saharan African countries. The formal sector sample was winsorized to replace outliers at the top 1% of the size distribution

cross-sectional age distribution of firms. While not as definitive as tracking firm cohorts over time, the cross-sectional approach provides suggestive insights into firm growth and selection over the life cycle.

Figure 4 shows that the young and economywide firm size distributions closely align in Ghana and Rwanda, whereas Cameroon exhibits a notably lower share of employment in large young firms. In Ghana and Rwanda, the similarity between the size distributions suggests limited business dynamism, with young firms struggling to grow and unproductive firms persisting rather than exiting the market. In contrast, the divergence between young and economywide firm size distributions in Cameroon suggests the possibility of healthier firm growth and selection over time. However, as indicated by the dotted lines in the figures, a significant share of employment at the top of Cameroon's economy-wide distribution is concentrated in a few firms that are over 30 years old. Due to the cross-sectional nature of the data, it is unclear whether these large, old firms grew gradually over time or were large from inception, possibly as state-created enterprises. [11].

[11]The data confirms that, at the time of being surveyed, most of these large and old firms

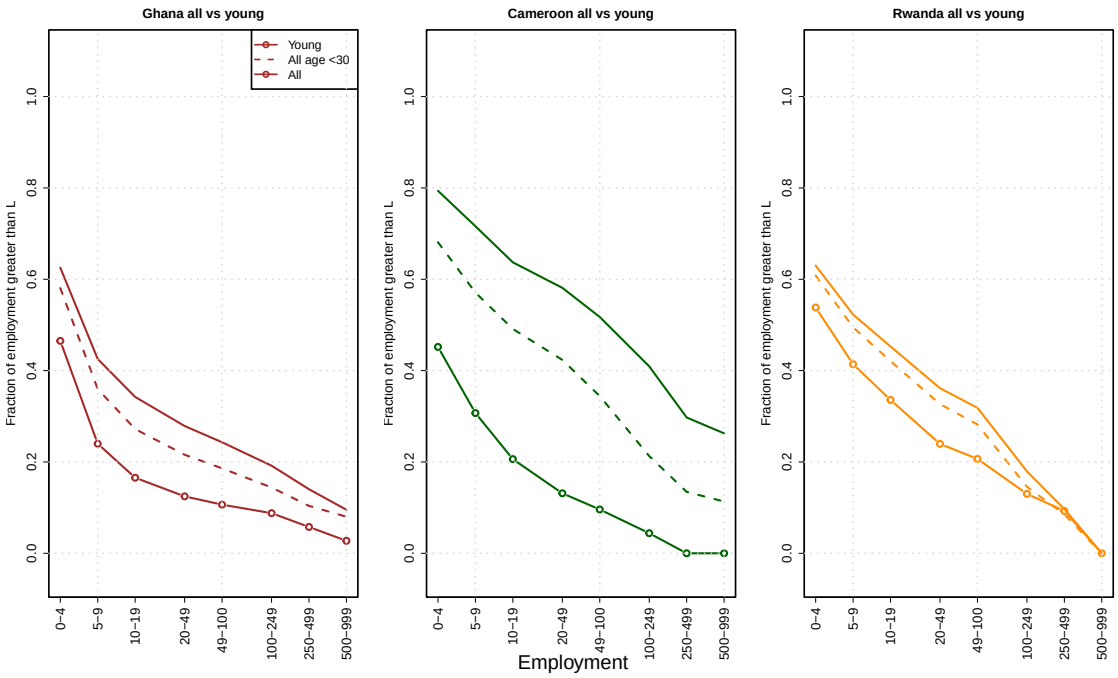

FIGURE 4. Formal Sector's Establishment and Employment-Based Size Distributions: SSA vs USA

The figure shows the employment-based size distribution for young and all manufacturing firms in Ghana (leftmost graph), Cameroon (middle graph), and Rwanda (rightmost graph). Both formal and informal firms are included.

In Figure 5, we compare the size distribution of young formal-sector firms with that of the overall formal-sector economy. Previously, Figure 2 demonstrated that the formal firm size distribution in SSA closely resembles that of the U.S. This similarity raises an important question: Do firms in SSA grow significantly as they age, or is firm size largely determined at entry, as observed in economies with firm growth distortions and entry barriers? The evidence in Figure 5 supports the latter view, suggesting that firm growth over time is limited. However, Cameroon remains a partial exception, consistent with earlier findings. Notably, its economywide formal-sector distribution is disproportionately influenced by a few large, older firms. Furthermore, the comparison between young firms and all formal firms younger than 30 years suggests a weaker pattern of firm dynamism.

In summary, our analysis of firm and employment-based size distributions in SSA highlights key differences from those in an advanced economy like the U.S. Most firms

are not government-owned. However, given their age and the precedent of a large footprint of the state in Cameroon, these firms are likely privatized state-owned enterprises. For more information on the privatization efforts started in Cameroon in the mid-1980s, see the technical annex to the Privatization and Technical Assistance Project between the World Bank and Cameroon, accessible at https://documents.worldbank.org/pt/publication/documents-reports/documentdetail/420681468769483724/cameroon-privatization-and-private-sector-technical-assistance-project

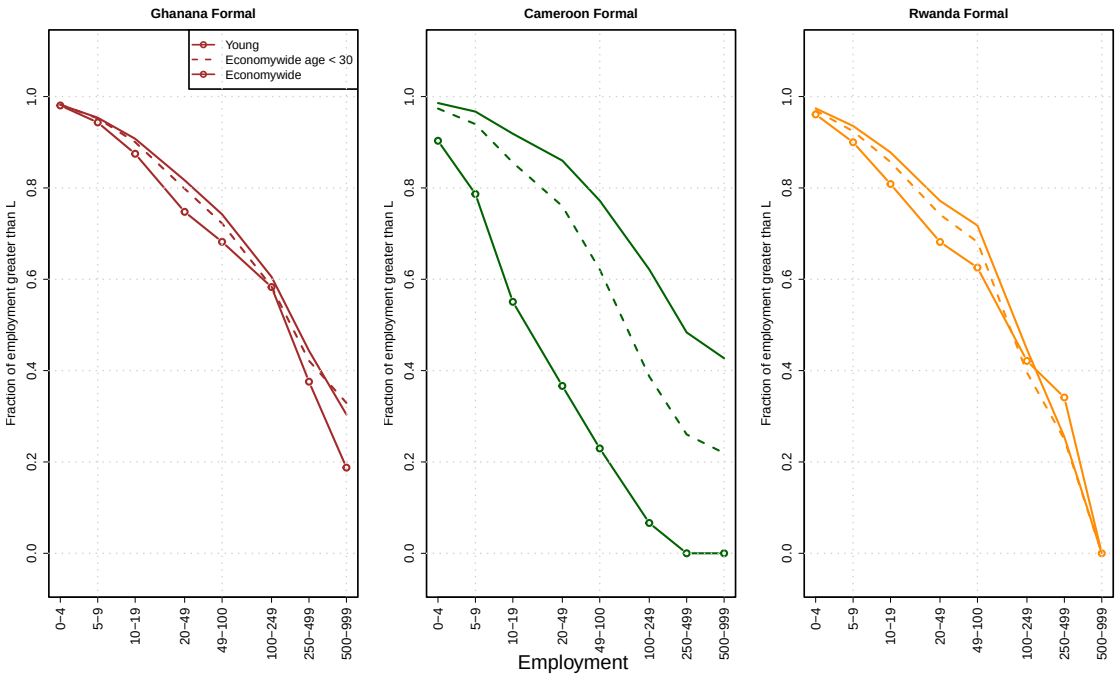

FIGURE 5. Economywide vs Young Formal Firms' Employment-Based Size Distributions

The figure shows the employment-based size distribution for young and all manufacturing firms in Ghana (leftmost graph), Cameroon (middle graph), and Rwanda (rightmost graph). Only formal firms are considered.

in SSA are micro-enterprises, and employment is less concentrated at the top of the distribution. These patterns largely stem from the prevalence of informal producers. However, the size distribution of formal firms—those that maintain formal bookkeeping practices and are registered with government agencies—closely mirrors that of the U.S. Crucially, comparing the size distributions of young firms and all formal firms suggests that this similarity does not result from organic firm growth and selection, as in the U.S. Instead, it reflects allocative distortions and entry barriers that shape the firm size distribution early on, limiting subsequent expansion.

5. A Model of Firm Dynamics with Formal and Informal Production

This section presents a model of firm dynamics to help us identify the underlying distortions in SSA's economies that rationalize the patterns in their firm size distributions and quantify their aggregate implications. Our interpretation of these patterns is that they reflect the equilibrium choices of firms in response to distortions in the environment, either in the form of barriers to formal firm creation or distortions to resource allocation among formal firms, without which their economies would have looked like the US. While the formal sector in the low-income countries under study exhibited some dynamism

compared to its informal counterpart, it still fell short of the dynamism of firms in advanced economies. This leaves a role to be played by productivity-dependent distortions, discouraging firm growth and misallocating resources across firms. Entry barriers, in turn, limit the incentives of firms to enter the formal sector and magnify the effect of idiosyncratic distortions.

The model features endogenous margins of adjustment that have first-order effects on the firm size distribution: an endogenously determined size of the formal and informal sectors, the entry and exit of firms in each sector, and innovation technology for formal firms. It will be calibrated to replicate the small size of informal production in the US and statistics on the size distribution of formal firms. Then, equipped with the undistorted model, we shall introduce idiosyncratic distortions, modeled as productivity-dependent revenue taxes, the properties of which we shall extract from Ghana's firm-level data, and a barrier to formal sector entry, which we'll parameterize to replicate Ghana's average firm size relative to the US. Last, we shall use the entire firm size distribution as an empirical counterpart to validate the model's attributes and quantify the macroeconomic implications of the underlying distortions.

5.1. Preferences and Demand System

Consumers derive utility from baskets of formal and informal goods, which they combine through a Constant Elasticity of Substitution demand system. Formally:

$$Q = \left[Q_1^{\frac{\lambda-1}{\lambda}} + Q_2^{\frac{\lambda-1}{\lambda}} \right]^{\frac{\lambda}{\lambda-1}}$$

where Q is the final good and Q_1 and Q_2 are formal and informal good baskets that go into the production of the final good.[12] In our calibration, parameters will be set so that the equilibrium expenditure and employment share in the informal sector are low, targeting values of the informal sector share in the U.S. economy. When distortions are identified and fed into the model, the elasticity of substitution λ will be a key determinant of the strength of the response of the informal sector to the distortions.

Formal and informal good baskets, in turn, are produced according to a Constant Elasticity of Substitution aggregation of a continuum of differentiated varieties, given by:

$$Q_i = \left[\int \left[q_i^d(\omega) \right]^{\frac{\theta-1}{\theta}} M_i(e^\omega)\, de^\omega \right]^{\frac{\theta}{\theta-1}} \tag{1}$$

where, hereafter, $i = 1$ and $i = 2$ shall denote the formal and the informal sectors respec-

[12] One could have directly defined Q to be aggregate consumption, where the aggregation is done at the level of the household. This is isomorphic to our specification of Q as a good produced by a final good aggregator.

tively. In turn, $M_i(e^{\omega})$ stands for the number of firms in sector i producing a variety with idiosyncratic productivity e^{ω}, to be determined endogenously in equilibrium.

Assuming a perfectly competitive representative producer of the aggregate final good and the aggregate basket of informal and formal varieties leads to the following demand functions for a given variety in a sector:

$$p_1(\omega) = P_1^{\frac{\theta-\lambda}{\theta}} Q^{\frac{1}{\theta}} q_1(\omega)^{\frac{-1}{\theta}} \tag{2}$$

Moreover, setting the final good as the numeraire, the aggregate and sectoral price indices are given by:

$$1 = \left[P_1^{1-\lambda} + P_2^{1-\lambda}\right]$$

$$P_i = \left[\int p_i(\omega)^{1-\theta} M_i f(e^{\omega})\, de^{\omega}\right]^{\frac{1}{1-\theta}} \tag{3}$$

5.2. Production of Differentiated Varieties

Consider now the producers of intermediate varieties of each good. Production is done under monopolistic competition according to the following technology:

$$y_i(\omega) = (A_i)^{\frac{1}{\theta-1}} (e^{\omega})^{\frac{1}{\theta-1}} l_i(\omega) \tag{4}$$

where A_i stands for a sector-wide productivity term and e^{ω} denotes the idiosyncratic productivity of intermediate producers. Labor is the sole factor of production. In what follows, we shall assume that A_i is constant over time and common across all firms in sector i, while there is a stochastic process of idiosyncratic productivity to be specified below.[13] This aspect of the stochastic process differentiates the production possibilities between formal and informal producers.

Profit maximization under monopolistic competition involves solving the following optimization problem (we present it for a formal sector producer with the understanding that an equivalent problem is solved in the informal sector):

$$max_{l_1}\left\{(1-\tau_{\omega}) P_1^{\frac{\theta-\lambda}{\theta}} Q^{\frac{1}{\theta}} (e^{\omega})^{\frac{1}{\theta}} A_1^{\frac{1}{\theta}} l_1(\omega)^{\frac{\theta-1}{\theta}} - w l_1(\omega)\right\}$$

where we have plugged in the demand function solved in equation 2.

[13] For formal firms, there is an innovation technology on which firms can make investments to choose the path of idiosyncratic productivity alongside idiosyncratic shocks. For informal firms, firm dynamics are deterministic, subject to exogenous exit shocks.

The idiosyncratic distortion, one of the types of distortion we consider, is reflected as a revenue tax in the producer's profit maximization problem. The key feature of this tax (interchangeably referred to as wedge) is that it is idiosyncratic and systematically related to the firm's idiosyncratic productivity e^{ω}, consistently with the misallocation literature.[14] Formally, we characterize this relationship with the following function:

$$(1-\tau_{\omega}) = (e^{\omega})^{\frac{-\gamma}{\theta-1}} \tag{5}$$

The key parameter governing the elasticity between *TFPR*, the measure of distortions, and *TFPQ*, the idiosyncratic productivity, is given by γ_i. Our calibration section discusses how we estimate this elasticity in the Ghanaian firm-level data.

Optimization leads to the following expressions for labor demand and variable profits:

$$l_1(\omega) = \left(\frac{\theta-1}{\theta}\right)^{\theta}\left(\frac{Q}{w^{\theta}}\right)P_1^{(\theta-\lambda)}e^{\omega}A_1(1-\tau_{\omega})^{\theta} \tag{6}$$

$$\pi_1^{\gamma}(\omega) = \frac{(\theta-1)^{\theta-1}}{\theta^{\theta}}P_1^{(\theta-\lambda)}\frac{Q}{w^{\theta-1}}e^{\omega}A_1(1-\tau_{\omega})^{\theta} \tag{7}$$

5.3. Firm Dynamics, Entry, and Exit in the Formal Sector

Our characterization of the formal sector features the first-order margins of adjustments through which distortions affect micro and macro-level outcomes. These are an endogenous entry and exit of firms into the formal sector and an innovation technology that allows for an endogenous component of firm dynamics.

Endogenous firm dynamics in the formal sector arise from a process of technological upgrading and downgrading similar to that in Atkeson and Burstein (2010). Specifically, a firm with current productivity e^{ω} can upgrade to $e^{\omega+\Delta}$ with probability $q_t(\omega)$ and can downgrade to $e^{\omega-\Delta}$ with probability $(1-q_t(\omega))$. The expected growth rate, given by $q_t(\omega)$, is endogenous, as firms can allocate resources to innovation activities. The variance of the shock process (Δ), on the other hand, is exogenous. The labor-denominated cost for attaining the desired probability $q_t(\omega)$ is given by

$$\chi(q_t,\omega) = e^{\omega}\times\eta\left(e^{\phi q_t}-1\right).$$

Notice that the innovation cost is scaled by the entrepreneur's current productivity. This is an important assumption that allows the model to be consistent with the innovation patterns of large firms in the US, our target economy for the calibration. The scale param-

[14] See, for instance (Bento and Restuccia 2017) and (Fattal-Jaef 2022) for cross-country estimates of the relationship between idiosyncratic distortions and idiosyncratic productivity

eter η and the elasticity parameter ϕ will be calibrated to replicate the properties of the size distribution and the life cycle of firms in the U.S.

The value of an operating formal firm with current productivity e^{ω} is

$$v_{1,t}^{o}(\omega) = \max_{q_t(\omega)} \left\{ \begin{array}{c} \pi_{1,t}^{v}(\omega) - w_t \chi(q_t, \omega) - w_t f_c \\ +R_t(1-\delta_1)\left[q_t(\omega) v_{1,t+1}\left(e^{\omega+\Delta}\right) + (1-q_t(\omega)) v_{1,t+1}\left(e^{\omega-\Delta}\right)\right] \end{array} \right\} \quad (8)$$

where $\pi_{1,t}^{v}(\omega)$ is the indirect variable profit function under optimal factor demands given prices and distortions, defined in equation 7, f_c denotes the labor-denominated fixed cost of production, and δ stands for the exogenous exit probability. While there is an exogenous exit shock, the presence of fixed costs gives rise to an endogenous motivation for the least profitable businesses to exit the market.

The value of the formal firm is given by:

$$v_{1,t}(\omega) = max_{\iota_{1,t}(\omega)} \left\{ v_{1,t}^{o}(\omega), 0 \right\},$$

where $\iota_{1,t}(\omega)$ encodes the firm's exit decision, equal to one if it operates and equal to zero if it exits.

The first-order condition with respect to the innovation choice yields

$$w_t \phi \mu e^{\phi p(\omega)} = R_t(1-\delta)\left[v_{1,t+1}\left(e^{\omega+\Delta}\right) - v_{1,t+1}\left(e^{\omega-\Delta}\right)\right].$$

Firms choose the probability of a technological upgrade to equate the marginal cost of innovation efforts with the gain in firm value. To the extent that idiosyncratic distortions are productivity dependent, as we are assuming in equation 5, these distortions will have a direct contribution to the rate of return to innovation expenses, in addition to general equilibrium effects on wages and final demand. Notice that entry barriers, which we have not introduced yet, have no direct effect on the firms' innovation decisions (though they will still have an indirect contribution through general equilibrium forces). Lastly, general equilibrium forces shaping the relative price between formal and informal goods and the wage rate will also impact the firm's incentives to innovate.

We turn now to discuss the entry technology into the formal sector. As standard in models of firm dynamics, we assume there is an infinite pool of potential entrants considering incurring a labor-denominated entry cost of f_e to enter the market. At the time of confronting this cost, there is no knowledge of the idiosyncratic productivity, but there is knowledge about the distribution $\Gamma(\omega)$ from which productivity is drawn. In equilibrium, a free-entry condition holds equating the cost of entry with the expected net present value of profits. In the distorted economy, the entry cost is inflated by an entry barrier τ^e, which is the second type of distortion we consider for rationalizing the

informality in the data. The free-entry condition in this context is given by

$$w_t f_e \left(1+\tau^e\right) = R_t \left(1-\delta\right) \int v_{1t+1}\left(\omega\right) d\Gamma\left(\omega\right). \tag{9}$$

Notice that we are assuming a one-period time to build between the payment of the entry cost and the actual entry.

5.4. Firm Dynamics, Entry, and Exit in the Informal Sector

The informal sector's firm dynamics and size distribution are entirely determined by the properties of the productivity distribution across firms. Unlike the richness of the business dynamics in the formal sector, our empirical analysis shows that informal producers are primarily microenterprises with little room for growth. Thus, we replicate the salient properties of the informal sector's size distribution by specifying a deterministic life-cycle path of idiosyncratic productivity subject to exogenous exit shocks. The economywide size distribution, in turn, is shaped by equilibrium variables in the model—specifically, the share of informal employment and the distribution of innovation expenses across formal firms. Since these two margins adjust in response to entry barriers and idiosyncratic distortions, they generate testable implications that can be validated with data.

Formally, the informal firms' productivity dynamics are given by a deterministic growth process where the exogenous growth and exit rates, μ and δ_2, jointly shape the age and size distributions across firms (Luttmer 2010). Thus, the idiosyncratic productivity of a firm with age a is given by:

$$e^{\omega(a)} = e^{\mu a}$$

and the fraction of active informal firms of age a is given by:

$$f\left(a\right) = e^{-\delta_2 a}$$

As said earlier, the production technology of informal firms adopts the same shape as that for formal producers, given in equation 4. Therefore, the outcome of the static optimization problem for the informal producer is given by the same expressions characterizing labor demand and variable profits in the formal sector. The sole difference between the two is that we abstract from idiosyncratic distortions across informal firms.[15]

Entry into the informal sector also entails a labor-denominated entry cost but is free from any entry barrier. The free-entry condition into the informal sector, then, is governed by:

[15] Our rationale for abstracting from distortions across informal producers is that the scope for allocative efficiency gains is limited, given the dramatic concentration of production among very small producers. Our interpretation is that the size distribution of informal firms reflects poor productivity or entrepreneurial ability rather than a manifestation of an idiosyncratic distortion.

$$wf_{e2} = \int \pi_2^v(a)\, e^{-(\rho+\delta)a} da$$

where f_{e2} is the entry cost into the informal sector, ρ is the subjective discount factor, and $e^{-(\rho+\delta)a}$ characterizes the expected productivity of the firm over its life-cycle. As for the formal sector, f_{e2} captures non-distortionary elements of setting up an informal firm, such as the opportunity cost of implementing the business idea. Naturally, this calibration strategy will deliver a significantly lower cost of entry into the informal sector.

5.5. Households

In addition to the static problem of choosing formal and informal expenditure, the representative household confronts the dynamic decision of allocating its wealth into consumption and savings, and how to split the savings into a risk-free bond and the portfolio of formal and informal firms in the economy. Given an initial endowment of formal and informal firms $M_1(0)$ and $M_2(0)$, the household's dynamic problem is formally given by[16]:

$$\int e^{-\rho t} log\left[c(t)\right] dt,$$

subject to

$$\dot{a}(t) = \begin{array}{c} r(t)\, a(t) + w(t)\, L + M_1 \int \pi_1^v(a,t)\, f(a)\, da + \\ M_2 \int \pi_2^v(a,t)\, f(a)\, da - f_{e1} M_{e1}(t)\, w(t) - f_{e2} M_{2e}(t)\, w(t) + T \end{array}$$

$$\dot{M}_1 = -\delta_1 M_1 + M_{e1} \tag{10}$$

$$\dot{M}_2 = -\delta_2 M_2 + M_{e2} \tag{11}$$

where T stands for the transfer to/from the household that balances the budget from the collection of revenue from entry barriers and idiosyncratic distortions.[17] The Euler equation ruling the incentives to accumulate assets implies that, in the stationary equilibrium, $\rho = r$.

[16] Notice that informal firms' dynamics and the household's dynamic optimization problem are presented in continuous time, whereas the format firms' value functions and free entry conditions are introduced under discrete time notation. This abuse of notation is innocuous in light of our focus on a stationary equilibrium.

[17] The lump-sum transfer is how we implement the idea that we do not interpret τ^E and $[1 - \tau_w]$ literally as taxes that drag resources away from the economy beyond the distortion they generate. In our model, output and consumption are reduced only to the extent that entry barriers and allocative distortions affect aggregate productivity.

5.6. Equilibrium

A stationary competitive equilibrium in the model consists of (i) consumption, firm entry in each sector, and saving decisions of the household $\left[c, \dot{a}, M_{e1}, M_{e2}, \dot{M}_1, \dot{M}_2\right]$; (ii) labor demands, prices, and variable profits for each variety $\left[l_i(\omega), p_i(\omega), \pi_i(\omega)\right]$; (iii) demand functions for intermediate inputs and final output, and sectoral price indices $\left[y_i(\omega), Y_i, P_i\right]$; (iv) the age distributions of informal firms, the schedule of idiosyncratic distortions, and the entry barrier: $\left[f_2(a), [1-\tau_\omega], \tau^E\right]$; (v) laws of motion for the productivity distribution of formal firms and the number of informal producers; (vi) innovation and exit decisions of formal firms $q(\omega), \iota(\omega)$; and (vii) wages and interest rates such that: (a) given wages, interest rates, and the laws of motion for the distribution of formal and informal producers (i) solves the household's optimization problem; (b) given wages and idiosyncratic distortions, (ii) solves each variety producer's profit maximization problem and (vi) solves the value function of a formal firm; (c) given the prices of formal and informal varieties, (iii) solves the final good's representative producer profit optimization problem; (d) the free entry conditions in each sector hold; (e)the labor market clears; and (f) net asset demand is equal to zero.

The labor market clearing condition in the equilibrium is given by:

$$L = L_{p1} + L_{fc} + L_I + L_{p2} + f_{e1}M_{e1} + f_{e2}M_{e2}. \tag{12}$$

where L_{pi} stands for the labor demand devoted to the production of formal and informal varieties, L_{fc} and L_I denote the labor demand associated with fixed and innovation costs in formal firms, and the remaining elements capture the labor demand allocated to the entry of new formal and informal producers.

The law of motion for the distribution of formal firms, in turn, is given by the following equation:

$$\begin{aligned} M_{t+1}(\omega') = (1-\delta) q_t\left(\omega' - \Delta\right) M_t(\omega' - \Delta) \\ + (1-\delta)\left[1 - q_t\left(\omega' + \Delta\right)\right] M_t(\omega' + \Delta) + (1-\delta) M_{e,t} \Gamma\left(\omega'\right). \end{aligned} \tag{13}$$

The expression establishes that a fraction $(1-\delta)\, q_t\left(\omega' - \Delta\right)$ of firms with productivity less than or equal to $\left(\omega' - \Delta\right)$ survive the exogenous exit shock and transition to a productivity level less than or equal to ω'. A fraction $(1-\delta)\left[1 - q_t\left(\omega' + \Delta\right)\right]$ of the mass of firms with productivity between ω' and $\left(\omega' + \Delta\right)$ survives the exit shock and jumps downward to have productivity less than or equal to ω'. There is also an inflow of new firms into this group, which is given by the mass of entrants, a fraction $\Gamma\left(\omega'\right)$ of which will feature productivity less than or equal to ω'. The endogenous exit will be driven by the mass of firms that transition downwards from the productivity cutoff, $(1-\delta)\, q_t\left(\underline{\omega} + \Delta\right) M_t\left(\underline{\omega} + \Delta\right)$.

6. Quantitative Analysis

We turn now to the quantitative analysis of the role of entry barriers and idiosyncratic distortions in accounting for the extent of informal production and the shape of the size distribution in Sub-Saharan Africa. We first discuss the model's calibration, present the strategy for identifying distortions, and then move on to quantifying the micro- and macro implications.

6.1. Calibration

Adopting the US as our undistorted benchmark, we calibrate the model's parameter values to match salient establishment-level statistics in the US manufacturing and non-agricultural sectors.

A key set of parameters to calibrate includes the relative entry costs, f_{e1} and f_{e2}, and the relative sector-wide productivity levels, A_1 and A_2. These parameters directly influence the average firm sizes of formal and informal producers and the overall share of informal employment in the economy. Thus, we calibrate them to match these empirical moments in an undistorted economy like the U.S.

For informal employment shares, we target the share of unincorporated self-employment relative to total non-agricultural employment. According to the BLS' Current Population Survey Bureau of Labor Statistics (2025), this share averaged 6

Data on the average firm size in the informal sector is not publicly available for the U.S. However, the BLS reports the distribution of paid employment among unincorporated self-employed individuals by firm size. As shown in Bureau of Labor Statistics (2016), this distribution is concentrated in micro-enterprises, resembling patterns observed in Sub-Saharan Africa. Given this similarity, we adopt Ghana's informal average firm size of 4 as the model's informal sector target.

For the formal sector, we set the average firm size to match the average establishment size in U.S. manufacturing, which was 45 workers in 2022. Table 3 reports the calibrated parameter values.

The parameters in the innovation cost function for the formal sector, η and ϕ, are calibrated to replicate the life-cycle growth and the employment share of the 10 largest firms in the U.S. manufacturing sector. The exogenous exit rate in the formal sector, δ_1, determines the exit rate of large establishments and is calibrated to match the observed exit rate among large producers in the U.S. Meanwhile, the growth and exit rates in the informal sector, μ and δ_2, are set to replicate the small employment share of large firms in Ghana's informal sector size distribution.

The final set of parameters to be calibrated in an undistorted economy pertains to the elasticity of substitution between formal and informal goods, as well as the elasticity of

TABLE 3. Parameter Values

Parameter	Values	Target
δ_1, δ_2	0.025 Exit rate large formal firms	
$\frac{\mu}{\delta_2}$	0.3	Employment share of the top 10 largest establishments in the informal sector = 20%
λ	3	Bachas et al. (2021)
θ	4	Hsieh and Klenow (2009)
ζ	4.5	Average size of entrant relative to incumbent U.S. manufacturing
η, ϕ	14.9,0.00035	Life-cycle growth and employment share of the top 10% largest establishments in the U.S.
$\frac{f_{e1}}{f_{e2}}, \frac{A_1}{A_2}$	1.3, 4.14	Average firm size formal relative to informal and informal employment share 6%
γ_1	0.286	Regression coefficient $log\,(TFPR)$ vs $log\,(TFPQ)$
τ^e	4.75	Average firm size in Ghana's manufacturing sector relative to the U.S.

Source: The average size, the employment concentration at the top 10 largest firms, and the average size ratio between 21-25 year old and 1 year old firms in U.S.' manufacturing are drawn from the BDS database for the year 2022, establishment-based version Bureau (2022). The average firm size in Ghana's informal sector is computed from the full establishment census. The $log(TFPR)$ and $log(TFPQ)$ are computed exactly as in (Hsieh and Klenow 2009), estimated from Ghana's National Industrial Census of 2003. The entry barrier τ^e is calibrated following the methodology in (Fattal-Jaef 2022).

substitution across varieties within a given goods basket. For the elasticity between formal and informal goods, we rely on estimates from the literature that examine substitution patterns between modern and traditional retail stores (Atkin, Faber, and Gonzalez-Navarro (2018)). According to Bachas, Gadenne, and Jensen (2023), these estimates typically range between 2 and 4. Based on this, we set a value of 3 in our model. Finally, for the elasticity of substitution across differentiated varieties within a given goods type, we adopt a standard value of 4.

6.1.1. Calibrating Distortions

We focus on two broad categories of distortions encompassing a wide range of policies and frictions studied separately: idiosyncratic wedges, which misallocate resources across firms, and entry barriers, which limit incentives to enter the formal sector. As discussed in the literature review, several studies have examined the role of particular policies and frictions resulting in resource misallocation. However, given our goal of understanding the origins of firm-size distribution patterns documented earlier, idiosyncratic distortions provide a parsimonious characterization of these policies, thus offering the opportunity to examine the full effect of allocative distortions on the firm size distribution.

Entry barriers, in turn, serve as a summary measure for forces restricting competition in an industry. These include registration costs for new firms, state capture by large incumbents that collude with the government to create barriers to entry, and other anti-competitive behaviors designed to hinder new businesses. In our model, the entry barriers and idiosyncratic distortions will shift the rate of return to entry in the formal sector and reallocate production to the informal sector, where the prospects for business dynamism are limited. Moreover, the endogenous exit and innovation channels in the formal sector add margins of adjustment that map into the size distribution of firms and their life-cycle growth patterns.

We appeal to firm-level data from Ghana, one of our sample countries, to identify

idiosyncratic distortions. We work with the second round of the Integrated Business Establishment Survey (IBES II), conducted by Ghana's Statistical Service, which collects information about the firms' value-added, wage bills, and capital stocks. The database covers the universe of establishments with 50 or more workers and a representative sample of businesses with a workforce below this threshold.

Equipped with this information, we identify idiosyncratic distortions exactly as in Hsieh and Klenow (2009). Concretely, the idiosyncratic distortion is defined as the demeaned value of the firm's Revenue Total Factor Productivity relative to its 4-digit industry average, given by:

$$TFPR_{i,s} \propto \left(\frac{P_{i,s}Y_{i,s}}{K_{i,s}}\right)^{\alpha_s} \left(\frac{P_{i,s}Y_{i,s}}{L_{i,s}}\right)^{1-\alpha_s} \tag{14}$$

Equation 14 defines $TFPR_{i,s}$ as proportional to the geometric average of the marginal revenue products of labor and capital. Following Hsieh and Klenow (2009), we calibrate 4-digit industry shares based on value-added shares of capital and labor in the US's manufacturing sector. Each firm's wedge is demeaned by the average $TFPR_s$ in the 4-digit industry it belongs to.

The paper focuses on the elasticity between the idiosyncratic distortion, $TFPR_{i,s}$, and the idiosyncratic productivity of the firm $TFPQ_{i,s}$, as the most relevant statistic of the distribution of distortions for the characterization of changes in the size distribution of firms relative to an undistorted benchmark. This elasticity has first-order effects on the firm size distribution through various channels: 1) resource misallocation from high to low productivity firms, 2) selection of producers due to endogenous exit, 3) number of formal establishments due to endogenous entry and exit, and 4) life-cycle growth of firms due to endogenous innovation expenses.[18]

To measure idiosyncratic physical productivity while lacking firm-level price information, we leverage the theoretical structure of the model to back out the price that would be consistent with the CES demand system of the model. This approach results in the following expression for the model-implied $TFPQ_{i,s}$

$$TFPQ_{i,s} \propto \frac{\left(P_{i,s}Y_{i,s}\right)^{\frac{\theta}{\theta-1}}}{K_{i,s}^{\alpha_s} L_{i,s}^{1-\alpha_s}} \tag{15}$$

[18] As a reminder, recall that our starting point is a calibrated characterization of the US economy and our goal is to assess how idiosyncratic distortions and entry barriers can turn such an economy into one resembling those in SSA. Focusing on idiosyncratic distortions not systematically related to firm-level fundamentals creates misallocation but does not affect entry, exit, and innovation channels. Thus, it ignores the implications of distortions on mechanisms that have a first-order effect on size distribution and aggregate productivity.

Having specified measures of idiosyncratic distortions and physical productivity, our next task is calibrating the productivity-elasticity of distortions specified in the distortion profile in equation 5. To do this, we run the following regression on our data:

$$log\left(\frac{TFPR_{i.s}}{\overline{TFPR_s}}\right) = \alpha + \gamma log\left(\frac{TFPQ_{i.s}}{\overline{TFPQ_s}}\right) + \varepsilon_i \tag{16}$$

We obtain an estimate of $\gamma = 0.286$ for Ghana's manufacturing sector, which we feed into equation 5 to parameterize the productivity-elasticity of distortions in the model. [19]

Finally, we calibrate the entry barrier to the formal sector, τ^e, to match the average firm size in Ghana relative to the U.S. Fattal-Jaef (2022) shows that productivity-dependent distortions reduce the equilibrium average firm size in the model. While this reduction aligns with cross-country differences in firm size, which tend to increase with economic development, the decline is significantly larger than what is observed in the data. This discrepancy suggests the presence of complementary distortions that offset the impact of productivity-dependent misallocation on firm size. Entry barriers serve this role by limiting entry and concentrating employment in larger firms. To account for this, Fattal-Jaef (2022) determines the entry barrier as a wedge in the free-entry condition, ensuring that, after incorporating productivity-dependent distortions, the equilibrium average firm size in Ghana matches the data.

We calibrate the entry barrier to match the average firm size of 46 workers, which equals the average firm size among formal producers in the data characterized in section 3. Implementing the procedure described above yields an entry barrier of $\tau^e = 4.75$

6.2. Quantifying Micro and Macro Implications of Distortions

We now turn to our main quantitative exercise: examining the micro and macro implications of distortions and evaluating their role in shaping firm size distribution patterns in Sub-Saharan Africa. Using our calibrated values, we introduce each distortion into the model and assess its impact individually.

Figure 6 depicts the employment and establishment-based size distributions in the US, Ghana, and the model with idiosyncratic distortions and entry barriers. The figure illustrates the model's success in capturing the establishment distribution and the shortcomings in replicating the right tail of the employment distribution in Ghana.

The model captures the reallocation of establishments from the formal to the informal sector, increasing from 30% in the undistorted equilibrium to 96% in the distorted one,

[19] The actual estimate of the elasticity is 0.436. However, we also account for the fact that there is productivity-dependent misallocation in the US, which is our efficiency benchmark. By subtracting the United States' estimate for the productivity elasticity of distortions, equal to 0.15, from the one for Ghana, we reach the value of 0.286.

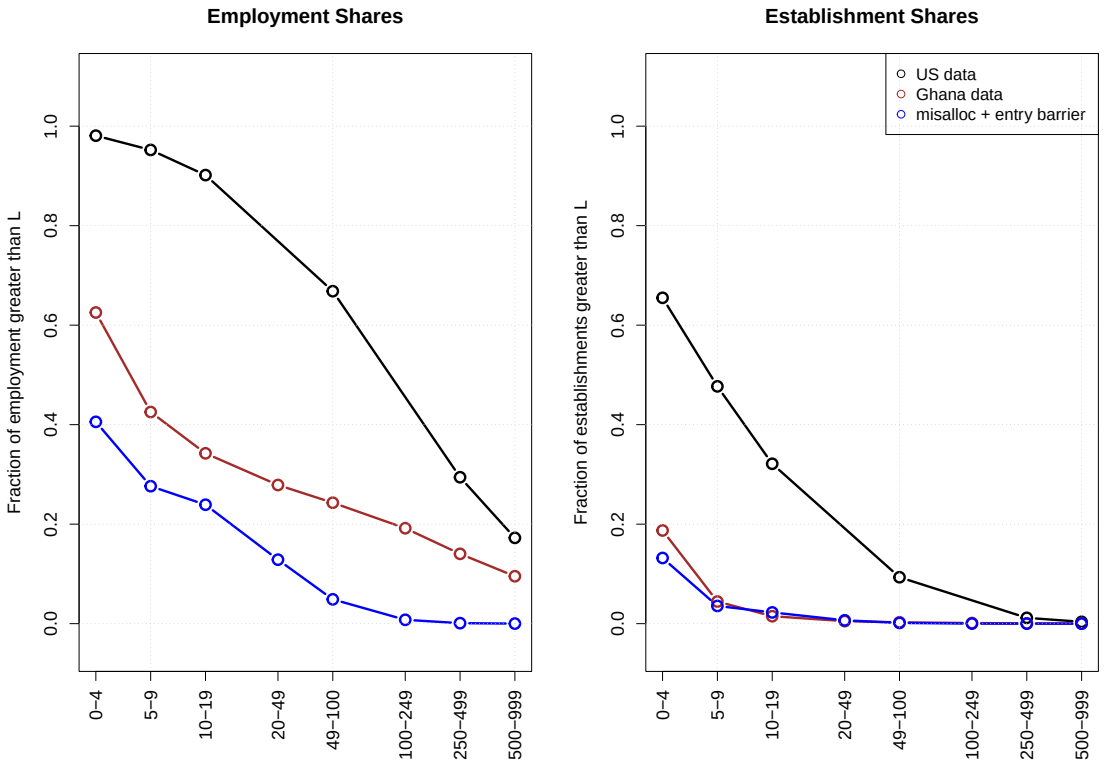

FIGURE 6. Establishment and Employment-Based Size Distributions: Distorted Model vs Data

The figure shows the employment (left panel) and establishment (right panel) size distribution in the US, Ghana, and the model with idiosyncratic distortions and entry barriers

closely matching Ghana's observed value of 97%. This shift occurs endogenously due to the disincentives to formalization created by the interaction of distortions, explaining the model's ability to replicate Ghana's establishment distribution.

The model with distortions replicates the rise in informal employment and the decline in employment concentration among large firms, but to a greater extent than observed in Ghana's data. To better understand the factors behind this excessive decline in the top employment share, consider Figure 7, which illustrates the employment and size distributions under each distortion individually.

When acting in isolation (red line in both panels of Figure 7), productivity-dependent idiosyncratic distortions cause a slight decline in formal production but significantly reallocate employment from larger to smaller firms. These distortions shift resources from more productive—and thus naturally larger—establishments to less productive ones, reducing innovation expenditures in formal firms. This, in turn, lowers their physical productivity, further driving employment away from the top of the size distribution. As a result, any actual policy or friction that generates the observed patterns of misallocation

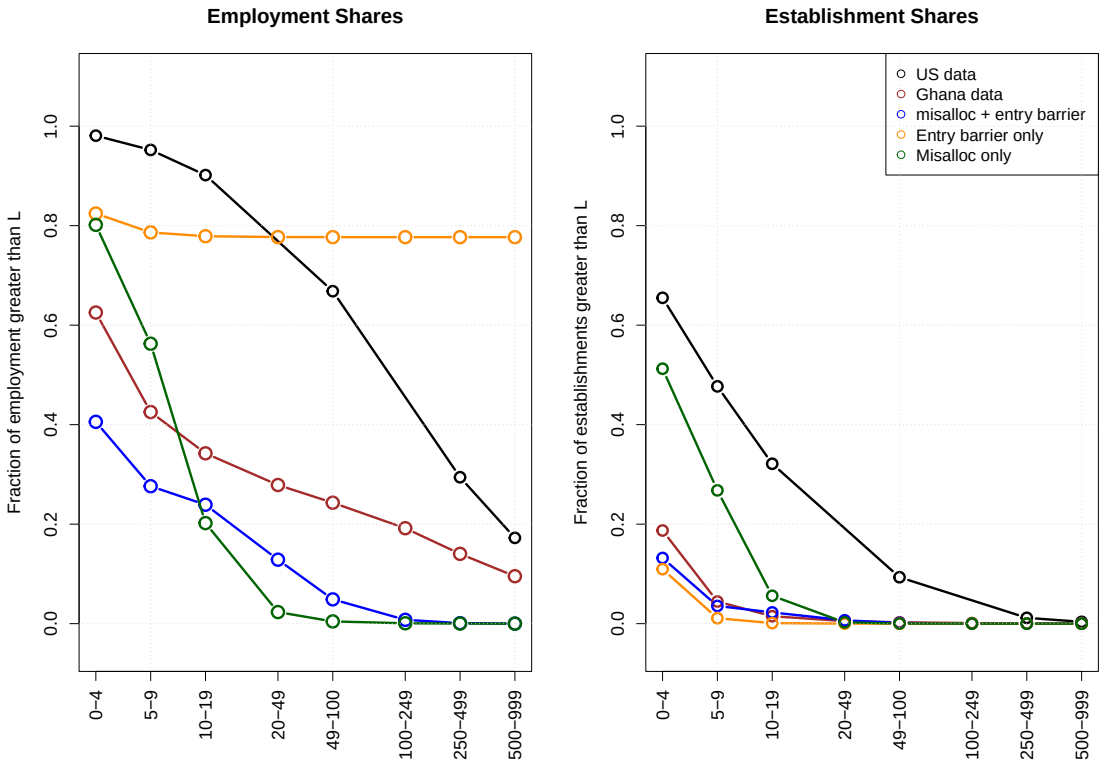

FIGURE 7. Establishment and Employment-Based Size Distributions: Single and Multiple Distortions vs Data

The figure shows the employment (left panel) and establishment (right panel) size distribution in the US, Ghana, and the model with idiosyncratic distortions and entry barriers, individually and interacting

will face similar limitations.

In contrast, entry barriers (orange lines in the figure) have the opposite effect on the size distribution. When acting alone, they restrict entry and concentrate employment among large incumbents, leading to an employment share at the top that far exceeds what is observed in Ghana, even surpassing the levels seen in the U.S.

When combined, idiosyncratic distortions and entry barriers partially offset each other's shortcomings, yielding a size distribution that better aligns with the data. However, the model still overstates the decline in employment shares among medium and large firms.

What explains the limitation? The reason is that the parsimonious characterization of misallocation, based on the estimated elasticity of *TFPR* vs *TFPQ* in the data, fails to capture firms that remain among the largest employers despite being among the least productive. However, when the sample is winsorized so that these unproductive yet large firms are assigned the size of the 99.9th percentile firm in the distribution, Figure 8 shows that the model—incorporating both entry barriers and idiosyncratic distortions—provides

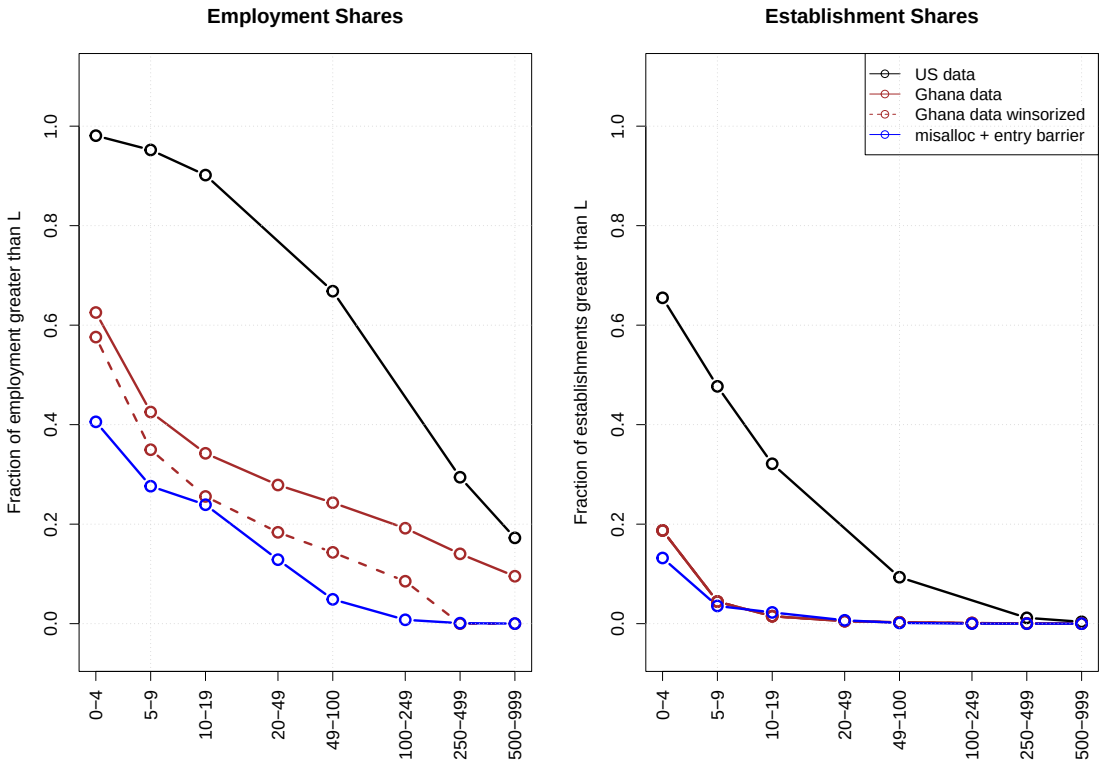

FIGURE 8. Establishment and Employment-Based Size Distributions: Single and Multiple Distortions vs Data

The figure shows the employment (left panel) and establishment (right panel) size distribution in the US, Ghana, and the model with idiosyncratic distortions and entry barriers, individually and interacting. The figure shows, in dotted lines, Ghana's size distribution under a winsorized sample at the 99.9th percentile of the distribution.

a much closer fit to the data.

Our final characterization of the micro-implications of the model with distortions refers to its predictions for the employment distribution of young formal firms. As discussed in Figure 5, the young formal establishments' distribution is close to the economy-wide one for formal firms in Ghana, suggesting little dynamism and life-cycle growth. Figure 9 evaluates this observation in the context of the model, assessing business dynamism in the model with distortions (left panel) and the undistorted economy (right panel).

Figure 9 shows that the model closely replicates the employment size distribution patterns for young firms and all firms in Ghana. Notably, it captures the similarity between the distributions of entrants and the overall economy. However, consistent with the baseline results, the model underpredicts employment shares at the upper end of the distribution. The right panel, which compares with a version calibrated to U.S. life-cycle dynamics, highlights that the interaction of distortions can account for the limited

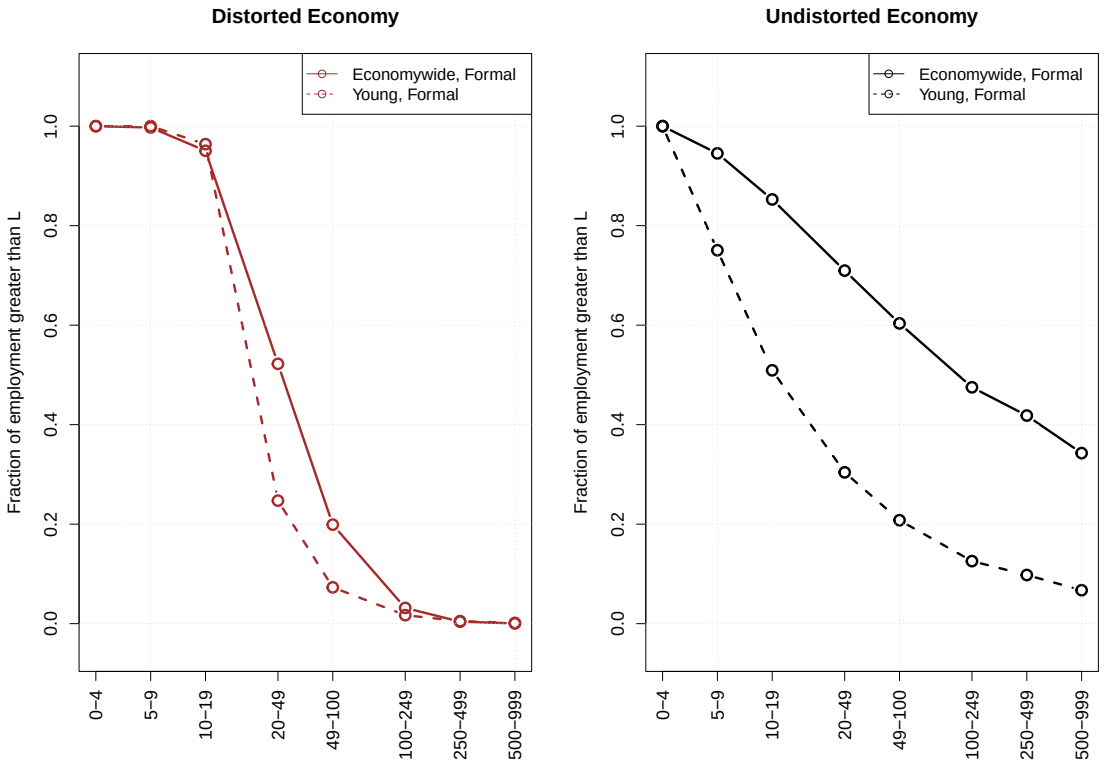

FIGURE 9. Employment-Based Size Distributions of Formal Firms in the Model with Entry Barriers and Idiosyncratic Distortions: Young vs Economywide

The figure shows the employment-based size distribution of formal firms in the model. The solid line depicts the economywide distribution, while the dashed lines illustrate the one for young firms (age less than or equal to 5) only. The left panel illustrates the size distributions in the economy with entry barriers and idiosyncratic distortions, while the right panel shows the undistorted economy.

business dynamism observed in Ghana.

We turn now to discussing the macroeconomic implications of the distortions just discussed. Table 4 reports aggregate variables of interest. We find that the combined effect of the entry barriers and productivity-dependent idiosyncratic distortion generates a 20% decline in aggregate output. Given the lack of physical capital in the model and the exogenous nature of labor supply, such a decline is the implied contraction in Total Factor Productivity.[20]. Individually, each distortion would contract GDP in the long run by 12% and 11%. Notice, then, that the interaction of distortions mitigates their individual

[20]Notice that the macroeconomic effects of removing distortions, particularly idiosyncratic distortions, stem only from the stylized productivity-dependent distortion profile postulated in equation 5. In earlier work quantifying misallocation in Sub-Saharan Africa, Cirera, Fattal-Jaef, and Maemir (2019) find substantially higher efficiency gains from resolving misallocation. These higher gains are attributed to accounting for the full extent of dispersion in distortions in the data, as opposed to the productivity-dependent component only. Here, our focus is only on the productivity gains attributable to entry barriers and productivity-dependent distortions, which are the ultimate drivers of informality in the model

aggregate productivity costs, leading to a combined contraction that is lower than the sum of the individual ones

The response in the number of firms helps rationalize the mitigating effect of the interaction of distortions. Under the joint application of entry barriers and idiosyncratic distortions, the number of establishments increases the most. While the majority of these are in the informal sector, as shown by the substantial increase in informal Value Added, their provision of a differentiated variety translates into an increase, all else equal, in aggregate output under our CES demand system.

6.3. Summary from Quantitative Analysis

Our quantitative analysis showed that the joint consideration of productivity-dependent idiosyncratic distortions and entry barriers can account for the patterns of the firm size distribution in Sub-Saharan Africa. In particular, when controlling for outliers at the top of the distribution, the model's fit is very close to the data in terms of the overall matching of the shape of the distribution, the share of informal establishments, and the lack of dynamism of formal firms over the life cycle.

TABLE 4. Aggregate Implications of Entry Barriers and Idiosyncratic Distortions

	Entry Barrier and Idiosyncratic Distortions	Entry Barrier	Idiosyncratic Distortions
GDP	0.81	0.88	0.89
GDP formal	0.28	0.72	0.72
GDP informal	8.56	3.14	3.34
Innovation Employment Share	-6.85	-0.27	-5.33
Total Number of Firms	5.31	1.61	3.24
Number of Formal Firms	0.32	0.00	2.63
Formal Establishment Share	0.03	0.00	0.47

Source: The table reports aggregate, formal, and informal GDP, the total number of firms and the number of formal firms in the three versions of a distorted economy(entry barriers and idiosyncratic distortions, entry barriers only, idiosyncratic distortions only) relative to their respective values in the undistorted calibration. The table also shows the percentage difference in the innovation employment share across the distorted economies relative to the undistorted one and the formal establishment share corresponding to each allocation.

At the aggregate level, the identified distortions significantly affect Total Factor Productivity, although there is still a sizable unexplained productivity gap relative to advanced economies. The 20% reduction in aggregate output falls short of the 60% to 80% declines documented in earlier studies on misallocation. For instance, Cirera, Fattal-Jaef, and Maemir (2019) finds a 75% increase in *TFP* from resolving misallocations in Ghana, using similar data to ours for an earlier period. The origin of the higher gains in these studies lies in their focus on the full extent of misallocation in the data. In contrast, here we focused only on the productivity-dependent component. The unaccounted aggregate productivity gap can be accounted for by sector-wide productivity differences unrelated to distortions. For instance, our study focused only on the manufacturing sector, whereas SSA economies remain excessively reliant on low-productivity agriculture.

Our quantitative findings also shed light on the specific policies and distortions underpinning the observed misallocation. To rationalize the shape of the size distribution, our findings dictate that they should combine policies that limit market contestability through barriers to entry with policies that hamper the potential growth of talented businesses and entrepreneurs. While we do not pinpoint the policies driving the identified wedges, we contribute by highlighting the key characteristics that candidate distortions should have to replicate the data successfully.

7. Conclusion

This paper has examined the role of firm-level distortions in shaping the size distribution of establishments in Sub-Saharan Africa, focusing on Cameroon, Ghana, and Rwanda. Using comprehensive establishment-level datasets, we documented key differences between these economies and the United States, notably the predominance of micro-enterprises and the relative scarcity of medium- and large-sized firms. Our empirical analysis revealed that these patterns are closely tied to the prevalence of informal production and that business dynamism is weak.

To interpret these empirical regularities, we developed a model of firm dynamics with endogenous formal and informal sector choices, heterogeneous productivity, and firm-level innovation. By calibrating the model to Ghana, where detailed firm-level data, including value-added and factor inputs, are available, we identified two critical sources of distortions: idiosyncratic distortions affecting firm productivity and resource allocation, and barriers to entry into the formal sector. The interaction of these frictions proved essential for replicating the observed firm size distribution, particularly the high share of employment in micro-enterprises and the emergence of a large informal sector.

Our quantitative results highlight the significant macroeconomic costs associated with these distortions. We estimate that the joint effect of misallocation and entry barriers reduces aggregate TFP by approximately 20% relative to a frictionless benchmark. Importantly, our findings emphasize that models incorporating only one type of distortion fail to match the data, either overstating employment in large firms or creating an implausible concentration in the middle of the distribution. Only when both frictions are present does the model generate a size distribution resembling the one observed in the data.

This analysis contributes to the growing literature that seeks to understand the microeconomic roots of aggregate productivity differences across countries. It underscores the importance of addressing both misallocation within the formal sector and barriers to formality to foster business dynamism and productivity growth in low-income economies. Addressing both types of misallocation is central to generating the increase in formal jobs needed to absorb the large numbers of entrants in the labor market in Sub-Saharan Africa. Future work could extend our framework by incorporating richer data on financial fric-

tions, tax compliance, and firm networks, or by exploring the dynamic policy implications of reform scenarios that reduce these distortions. Addressing the frictions identified in this paper could yield substantial gains in economic performance and the formalization of entrepreneurial activity in Sub-Saharan Africa.

Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the author(s) used [ChatGPT and Grammarly] in order to [check grammar, consistency, and improve readability]. After using this tool/service, the author(s) reviewed and edited the content as needed and take full responsibility for the content of the publication.

References

Alfaro, Laura, Andrew Charlton, and Fabio Kanczuk. 2008. "Plant-size distribution and cross-country income differences." NBER Working Papers 14060, National Bureau of Economic Research.

Alvarez, Jorge, and Cian Ruane. 2019. "Informality and Aggregate Productivity: The Case of Mexico." *IMF Working Paper Series # 2019/257*.

Antunes, Antonio R., and Tiago V. de V. Cavalcanti. 2007. "Start up costs, limited enforcement, and the hidden economy." *European Economic Review* 51 (1): 203–224.

Atkeson, Andrew, and Ariel Tomás Burstein. 2010. "Innovation, Firm Dynamics, and International Trade." *Journal of Political Economy* 118 (3): 433–484.

Atkin, David, Benjamin Faber, and Marco Gonzalez-Navarro. 2018. "Retail Globalization and Household Welfare: Evidence from Mexico." *Journal of Political Economy* 126 (1): 1–73.

Bachas, Pierre, Lucie Gadenne, and Anders Jensen. 2023. "Informality, Consumption Taxes, and Redistribution." *The Review of Economic Studies* 91 (5): 2604–2634. https://doi.org/10.1093/restud/rdad095.

Bento, Pedro, and Diego Restuccia. 2017. "Misallocation, Establishment Size, and Productivity." *American Economic Journal: Macroeconomics* 9 (3): 267–303.

Bureau, U.S. Census. 2022. "Business Dynamics Statistics. Sector by Firm Size." https://www.census.gov/data/datasets/time-series/econ/bds/bds-datasets.html.

Cirera, Xavier, Roberto Fattal-Jaef, and Hibret Maemir. 2019. "Taxing the Good? Distortions, Misallocation, and Productivity in Sub-Saharan Africa." *World Bank Economic Review* 34 (1): 75–100.

Dabla-Norris, Ms. Era, Laura Jaramillo, Frederico Lima, and Alexandre Sollaci. 2018. "Size Dependent Policies, Informality and Misallocation." IMF Working Papers 2018/179, International Monetary Fund. https://ideas.repec.org/p/imf/imfwpa/2018-179.html.

D'Erasmo, Pablo N., and Hernan J. Moscoso Boedo. 2012. "Financial structure, informality and development." *Journal of Monetary Economics* 59 (3): 286–302.

Fattal-Jaef, Roberto N. 2022. "Entry Barriers, Idiosyncratic Distortions, and the Firm Size Distribution." *American Economic Journal: Macroeconomics* 14 (2): 416–68.

Hsieh, Chang-Tai, and Peter J. Klenow. 2009. "Misallocation and Manufacturing TFP in China and India." *Quarterly Journal of Economics* 124 (4): 1403–1448.

Hsieh, Chang-Tai, and Benjamin A. Olken. 2014. "The Missing "Missing Middle"." *Journal of Economic Perspectives* 28 (3): 89–108.

Iacovone, Leonardo, Vijaya Ramachandran, and Martin Schmidt. 2013. "Stunted growth : why don't African firms create more jobs ?." Policy Research Working Paper Series 6727, The World Bank. https://ideas.repec.org/p/wbk/wbrwps/6727.html.

La Porta, Rafael, and Andrei Shleifer. 2014. "Informality and development." *Journal of Economic Perspectives* 28 (3): 109–26.

Bureau of Labor Statistics, U.S. Department of Labor. 2016. "BLS Spotlight: Self-Employment in the United States." https://www.bls.gov/spotlight/2016/self-employment-in-the-united-states/pdf/self-employment-in-the-united-states.pdf.

Bureau of Labor Statistics, U.S. Department of Labor. 2025. "Labor Force Statistics from Current Population Survey." https://data.bls.gov/dataViewer/view/timeseries/LNS12032192.

Lopez-Martin, Bernabe. 2019. "Informal Sector Misallocation." *Macroeconomic Dynamics* 23 (8): 3065–3098.

Luttmer, Erzo G. J. 2010. "Models of Growth and Firm Heterogeneity." *Annual Review of Economics* 2 (1): 547–576.

Poschke, Markus. 2014. "The Firm Size Distribution across Countries and Skill-Biased Change in Entrepreneurial Technology." IZA Discussion Papers 7991, Institute for the Study of Labor (IZA).

Sandefur, Justin. 2010. "On the Evolution of the Firm Size Distribution in an African Economy.."

Teal, Francis. 2023. "Firm Size, Employment and Value Added in African Manufacturing Firms: Why Ghana Needs Its 1%." *Journal of African Economies* 32 (2): 118–136.

Tsaedu, Kahsay G, Zhihong Chen, and Hana W Azmete. 2023. "Firm Size Distribution in African Manufacturing Firms: Revisiting the 'Missing Middle'." *Journal of African Economies* 32 (2): 137–150.

Tybout, James R. 2000. "Manufacturing Firms in Developing Countries: How Well Do They Do, and Why?." *Journal of Economic Literature* 38: 11–44.

Tybout, James R. 2014. "The Missing Middle: Correspondence." *Journal of Economic Perspectives* 28 (4): 235–236.

Ulyssea, Gabriel. 2018. "Firms, Informality, and Development: Theory and Evidence from Brazil." *American Economic Review* 108 (8): 2015–2047.

Ulyssea, Gabriel. 2020. "Informality: Causes and Consequences for Development." *Annual Review of Economics* 12 (1): 525–546.