Entry Barriers, Idiosyncratic Distortions, and the Firm Size Distribution

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Abstract

This paper studies the interaction between barriers to firm entry and distortions to allocative efficiency in a standard model of firm dynamics. We derive a strategy to infer entry barriers based on cross-country differences in the firm size distribution and idiosyncratic distortions. The inferred barriers resemble regulation-based indicators in advanced economies but are substantially higher in middle- and low-income countries. Regulation-based indicators cannot account for cross-country differences in average firm size and underestimate the aggregate productivity gains associated with their removal by up to 8 percent on average.

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1 Introduction

Distortions that discourage the entry of new firms and misallocate resources across incumbents have been postulated as drivers of the large differences in total factor productivity (TFP) across countries.[1] There are two limitations with existing attempts at quantifying the aggregate losses from entry barriers and idiosyncratic distortions. The first limitation concerns the measurement of entry barriers. While, under strong assumptions, one can identify idiosyncratic distortions as deviations from the optimal allocation (Hsieh and Klenow, 2009), there is no equivalent strategy to identify entry barriers based on firms' outcomes. As a result, researchers have to appeal to imperfect proxies of entry barriers, such as de jure indicators, leaving the assessment of the real burden of entry barriers an open question. The second limitation is the lack of an understanding of how entry and allocative distortions[2] interact in lowering TFP.[3] Are the productivity losses from idiosyncratic distortions magnified or mitigated by entry barriers?

In this paper, we address these limitations. We begin by characterizing the equilibrium implications of entry barriers and idiosyncratic distortions in a simple model of firm dynamics. We then derive a strategy to infer entry barriers based on cross-country differences in average firm size and cross-country estimates of idiosyncratic distortions. Next, in the context of a quantitative model featuring endogenous entry, exit, and innovation decisions by firms, we assess the productivity implications of each type of distortion.

Our inference strategy delivers sizable entry barriers in middle- and low-income countries that decrease to roughly zero in advanced economies. Compared with the widely used World Bank's Doing Business Indicator (DBI) cost

[1] Examples of concrete policies that manifest as allocative distortions include labor market regulations (Hopenhayn and Rogerson, 1993), financial frictions (Buera et al., 2011 and Midrigan and Xu, 2014), size-dependent policies (Guner et al. 2008; Gourio and Roys, 2014; and Bachas et al., 2019).

[2] We refer to idiosyncratic distortions and allocative distortions interchangeably.

[3] Boedo and Mukoyama [2012] stands out from the literature by jointly considering entry taxes, proxied by the World Bank's Doing Business Indicator, with a particular type of allocative distortion, firing taxes. We discuss our contribution relative to this paper in Section 2.

of starting a firm, our model's estimates closely follow this indicator in developed countries but are notably larger in less developed countries. Turning to the aggregate implications, we find that removing both types of distortions increases TFP by up to 50 percent in the long run and that the interaction between distortions mitigates each other's detrimental effect on TFP. Had there been no interaction, the productivity gains would have been up to 15 percent higher. Finally, we show that using the entry barriers from the DBI fails to replicate the cross-country average firm size distribution in the data and underestimates the aggregate TFP gains by 4 to 8 percent, on average.

Our analysis is conducted based on a standard model of heterogeneous firms featuring monopolistic competition and Constant Elasticity of Substitution (CES) aggregation. For our analytical characterization of the interaction between entry barriers and idiosyncratic distortions in the stationary equilibrium, we work with a simple version of the model where exit is exogenous and firm-growth is deterministic, as in Luttmer [2010]. This simplification allows for a closed-form characterization. For the quantitative analysis, we incorporate endogenous exit and endogenous innovation by firms, along the lines of Atkeson and Burstein [2010].

We begin by characterizing the effect of the interaction between entry barriers and idiosyncratic distortions in the economy's stationary equilibrium. We establish that: 1) entry barriers increase the average firm size, for any given value of idiosyncratic distortions, 2) idiosyncratic distortions that correlate positively with firm productivity decrease the average size, for any given level of entry barriers, and 3) the interaction between distortions mitigates each other's detrimental effect on aggregate productivity, and such mitigation operates through the number of firms in the economy.[4]While the effect of entry barriers and idiosyncratic distortions on average firm size has been studied before, the characterization of their interaction is a contribution to the literature.[5] We show that the introduction of idiosyncratic distortions raises the

[4]We are more concrete later in the paper. For now, by correlated distortion, we refer to the property in the data that idiosyncratic distortions, TFPR, have a positive elasticity with respect to the physical productivity of the firms, TFPQ.

[5]Examples of previous work studying the effect of entry barriers on average size and ag-

number of firms by a larger amount when entry barriers are in place, mitigating the negative effect of idiosyncratic distortions on TFP. Conversely, the negative effect of entry barriers on the number of firms is weakened when there are idiosyncratic distortions, and hence so is the decline in TFP.

We now explore the role of entry barriers in explaining the TFP differences across countries. At a superficial level, this may seem a non-starter, because the average firm size increases with income per capita in our data, which suggests that entry barriers are higher in richer countries. However, we show that firm size distributions are determined jointly by entry barriers and idiosyncratic distortions, and that entry barriers do play a role in explaining the cross-country TFP differences. When we introduce idiosyncratic distortions that are disciplined by firm-level data into our quantitative model, firms become too small on average relative to the data, which implies that the model is lacking an element that would push the average firms size higher: i.e., higher entry barriers for distorted, low-income countries.

We then build on this logic and develop a strategy to infer the level of each country's entry barrier. Essentially, we compute the magnitude of the entry barrier that, given the estimated elasticity between idiosyncratic distortions (TFPR) and idiosyncratic productivity (TFPQ), yields the equilibrium average size consistent with the data. Consider the example of Chile, one of the 21 countries in our collection firm-level database[6]. In Chile's manufacturing sector, firms employed an average of 94 workers in 2013, slightly below the average size of 118 in the US manufacturing sector, our benchmark of efficiency.[7] From the firm-level data, we estimate a productivity-distortion elasticity of 0.17.[8]

gregate productivity are Barseghyan and DiCecio [2011] and Boedo and Mukoyama [2012], while Fattal-Jaef [2018] and Bento and Restuccia [2017] are examples focusing on idiosyncratic distortions.

[6] Our data combine census-based information of manufacturing sectors in Latin America, South Asia, and sub-Saharan Africa with information on a subset of countries from the Amadeus database. More details are provided in Section 4.

[7] Since the Chilean manufacturing data cover the universe of manufacturing plants with 10 or more workers, we apply the same truncation to the US data to arrive at the average size of 118 workers in the US. We explain data coverage details for all countries in the data section.

[8] The actual estimate of the elasticity is 0.32. However, we take into account that there

According to our quantitative model, the idiosyncratic distortions should have reduced the average firm size to 36 workers, other things equal. To reconcile the equilibrium with the data, our model identifies a countervailing entry tax of 230 percent, about two times the entry cost of a firm in the U.S. Applying this procedure to all the countries in our data, we come up with a model-based distribution of entry barriers.

Equipped with the model-based estimates of entry barriers, our final contribution is to quantitatively assess the aggregate effects of entry barriers and idiosyncratic distortions. As stated earlier, the TFP gains from eliminating all distortions are substantial, ranging between 20 and 50 percent in the most distorted economies, and both distortions weigh in prominently in accounting for these gains. We complement these findings with two additional exercises. The first one quantifies the role of the interaction between the two distortions. We find that they mitigate each other's detrimental effect on aggregate productivity. Aggregate gains would be up to 15% higher if there were no such interaction. This result sheds light on the magnitude of the biases from evaluating each distortion in isolation, a common practice in the literature. The second exercise compares the model-based estimates of entry barriers with de jure measures, such as the World Bank's Doing Business Indicators. We find that using the DBI substantially underestimates the role of entry barriers, giving the mistaken perception that entry barriers are less important than idiosyncratic distortions in hindering economic development.

Prior to delving into the details of the paper, a couple of remarks are in order regarding the notion of entry barrier that we are adopting. Firstly, as explained earlier, we identify entry barriers as wedges in the free-entry conditions of the model. A first acknowledgment, then, is that our wedge will pick up actual entry barriers only to the extent that the model is a fair representation of reality. To alleviate concerns about misspecification, section 5.5 and appendix C discuss various extensions (heterogeneous markups, discount

is correlated misallocation in the US, our benchmark of efficiency. By subtracting the US' estimate of the distortion elasticity of 0.15 from the estimate for Chile, we reach the value of 0.17. We proceed in this fashion for all of the countries.

rate wedges, multiple sectors) and show that our results are robust. Secondly, while explicitly treating the entry barrier as a tax on the costs of firm entry, we think of it as encompassing the broad set of distortions that inefficiently increase the post-entry profitability of incumbents relative to a benchmark without such wedges. These could stem from the cost side of entry, as implied by the regulation-based World Bank's Doing Business' costs of starting a firm, from cross-country differences in the enforcement of such regulations, or from any other distortion to incumbents' average profitability that is not already captured in the productivity-dependent idiosyncratic distortion profile. While it is a limitation of the paper not to disentangle the various components of the entry wedge, its contribution lies in showing that the overall barriers to entry are far in excess of those implied by de jure based indicators in developing countries and that their associated TFP losses are substantially larger.

The remainder of the paper is organized as follows. Section 2 relates our work to the existing literature. Section 3 presents the model assuming exogenous exit and exogenous firm dynamics. This section also derives the theoretical insights underlying the identification strategy of entry barriers. Section 4 presents our data, explains the adjustments and selection criteria to construct the final sample, and computes the average size to income per capita elasticity. Section 5 conducts the quantitative analysis, presents the extensions of the simple theory to allow for endogenous exit and endogenous innovation, describes the calibration strategy, and performs the counterfactuals for computing the TFP gains. Section 6 concludes.

2 Related Literature

Our work is closely related to the large and growing literature on the effect of distortions on aggregate productivity. Salient work studying entry taxes includes Djankov et al. [2002], Poschke [2010],Barseghyan and DiCecio [2011], and Boedo and Mukoyama [2012]. Djankov et al. [2002] developed a methodology to measure entry barriers that ultimately became the backbone of the World Bank's Doing Business Indicators (DBI), the leading and most comprehensive database of de jure regulation of entry around the world. Poschke [2010], Barseghyan and DiCecio [2011], and Boedo and Mukoyama [2012] quantify the aggregate effects of entry barriers using the DBI. We contribute to this literature by providing a complementary metric of entry barrier that is based on equilibrium conditions and the average firm size across countries in the data. We show that our approach portrays a different perspective about the prevalence and costs of entry barriers in developing countries: these are more prevalent than suggested by de jure indicators and carry substantially higher TFP losses.

Peters [2019] also studies an economy where entry barriers are identified from firm-level outcomes. In Peters [2019], limits to the entry of competing firms and products interact with an endogenous determination of markups in shaping the life-cycle growth of firms in the model. The paper then identifies cross country differences in the entry costs of firms and products to reconcile life-cycle differences between Indonesia and the U.S. The main difference between this paper and ours is that we treat resource misallocation as exogenous. While ours is a restrictive assumption, it allows us to capture the full extent of misallocation in the economy. As we show below, accounting for the full extent of misallocation and its effect on the equilibrium average firm size is key for accurately identifying the required entry barriers in the economy that explain the observed average firm size distribution in the data. In some sense, our study could be thought of as providing an alternative estimate of entry barriers that can then be fed into Peters [2019]'s model to infer the resulting distribution of mark-ups and the associated implications for growth. The key is that such resulting distribution of mark-ups will only partially account for the dispersion in marginal revenue products across firms[9].

Similarly, Herrendorf and Teixeira [2011] characterize the interaction between entry barriers and rent extraction in a multi-sector growth model and

[9] Poschke [2010] also considers an interaction between entry barriers and mark-ups through a reduced-form relationship between the elasticity of substitution and the number of producers. In both studies, the interaction between entry barriers and some form of allocative distortion exists. However, as we argue below, accounting for the full extent of misallocation is essential for minimizing biases in the identification of entry tax rates.

identify entry barriers residually in order to match aggregate differences in productivity and capital to labor ratios across countries, taking into account aggregate and sector-specific distortions. While we share the residual approach to identification, our strategy differs in that entry barriers are disciplined by firm-level and idiosyncratic distortions, as opposed to economy and sector-wide frictions. In this way, our inference strategy fully captures average size differences across countries, leaving aggregate differences in productivity as a measure of the extent to which these distortions can account for productivity differences in the data.

Naturally, our work is also tightly connected with the literature on idiosyncratic distortions. From Hsieh and Klenow [2009], we inherit the inference strategy of idiosyncratic distortions as dispersion in marginal revenue products across firms within narrow sectors. An important feature of this strategy is the independence of the distribution of wedges from aggregate equilibrium variables in the economy. This independence, which emerges from defining misallocation as dispersion in marginal revenue products relative to the average return in the industry, enables us to estimate the elasticity between TFPR and TFPQ directly from the data and to devote the model's equilibrium condition to pin down the entry barrier. We differ from the earliest papers in this literature (Hsieh and Klenow 2009, Restuccia and Rogerson 2008), however, in two essential dimensions: 1) the consideration of dynamic margins of adjustment (entry, exit, innovation) that create an interaction between the firm size distribution and the properties of the idiosyncratic-distortion profile, and 2) the consideration of entry barriers in conjunction with resource misallocation[10]. While dynamic margins of adjustment to idiosyncratic distortions are also present in Hsieh and Klenow [2014], the joint consideration of idiosyncratic distortions and entry barriers, the derivation of a strategy to infer these from firm level data, and the characterization of the interaction between each

[10]The interaction between entry barriers and another particular type of allocative distortion, firing costs, is studied in Boedo and Mukoyama [2012]. In this study, however, entry barrier are read directly from the World Bank's Doing Business Indicators, whereas our goal is to identify these from firm-level data. For this goal, it is essential that we account for the full extent of misallocation in the economy.

distortion in accounting for TFP differences are contributions of our work.

Finally, our analysis is also closely related to Bento and Restuccia [2017], which also develops a model with entry, exit, and innovation to study the effect of idiosyncratic distortions on average firm size. While this paper also characterizes the effect of idiosyncratic distortions on average firm size and decomposes the effects into the various moving pieces, we differentiate from it in that we bring entry barriers to the analysis and leverage the full information in the distribution of average size across countries to discipline the identification of entry barriers. Once the interaction is considered, we not only provide a model-based measure as an alternative to de jure indicators of obstacles to entry, but we also show that the aggregate effects of idiosyncratic distortions are muted by the interaction with the underlying entry barriers.

3 Entry Barriers and Misallocation in a Simple Model of Firm Dynamics

In this section, we present a stylized general equilibrium model of deterministic firm dynamics and exogenous exit where we can sharply characterize the interaction between distortions in the equilibrium. The main purpose of this characterization is to demonstrate the balancing role that average firm size plays in absorbing the forces from the entry and allocative distortions. Such a role constitutes the backbone for the identification strategy of entry barriers that we pursue later in the paper. Furthermore, the simple model allows us to gain a better understanding of the macroeconomic consequences of the interaction between distortions. Are the TFP effects of one distortion mitigated or magnified by the presence of the other? Which distortion is creating more harm to the economy?

Technologies There is a single final good producer in the economy that operates in a perfectly competitive market and produces according to the following constant elasticity of substitution (CES) composite of intermediate inputs:

$$Y_t = \left[\int y_{d,t}(\omega)^{\frac{\theta-1}{\theta}} \, d\Omega_t(\omega) \right]^{\frac{\theta}{\theta-1}}, \tag{1}$$

where $\Omega_t(\omega)$ stands for the number of producers of differentiated variety ω and θ denotes the constant elasticity of substitution.

Profit maximization under perfect competition yields the following demand function for intermediate input ω:

$$y_{d,t}(\omega) = \left(\frac{p_t(\omega)}{P_t} \right)^{-\theta} Y_t. \tag{2}$$

The price index implied by the final good production technology is given by

$$P_t = \left[\int p_t(\omega)^{1-\theta} \, d\Omega_t(\omega) \right]^{\frac{1}{1-\theta}}.$$

In what follows, we adopt the final good as the numeraire in the economy, so we set $P_t = 1$ for all t.

The intermediate input varieties are supplied in monopolistically competitive markets and are produced by heterogeneously productive firms according to the following technology:

$$y(\omega) = [e^{\omega}]^{\frac{1}{\theta-1}} \, l_t(\omega). \tag{3}$$

Here $(e^{\omega})^{\frac{1}{\theta-1}}$ stands for the idiosyncratic productivity of the firm, which evolves exogenously according to a deterministic growth process to be characterized below.

Distortions and Static Optimization There are two types of distortions in the economy: entry barriers, which increase the monetary value of the sunk cost of entry that prospective entrants must confront to become active producers, and idiosyncratic distortions, which capture all sources of misallocation in an economy that manifest as wedges in the firms' optimal condition for factor demands. We first introduce the profile of idiosyncratic distortions, which have a direct contribution over the incumbent firms' decision problems, and postpone the specification of the entry barrier until the presentation of the technology for firm creation.

Following Buera and Fattal-Jaef [2018], the profile of idiosyncratic distortions adopts the following functional form[11]:

$$1 - \tau(\omega) = \left[(e^{\omega})^{\frac{1}{\theta-1}} \right]^{-\gamma}. \tag{4}$$

The key property of this schedule of revenue taxation is its productivity dependence, which is governed by the elasticity γ. Our motivation for focusing on the correlated component of idiosyncratic distortions is twofold. First, as we show below, we find strong support for this property of distortions in the data. More concretely, with varying degrees, all countries feature a positive elasticity between idiosyncratic distortions (TFPR) and physical productivity (TFPQ). Second, it is this property of the distortions that generates an active response of average firm size in the theory, hence allowing us to appeal to cross-country variation in misallocation to account for part of the cross-country variation in the firm size distribution. It is a well-established property of the canonical models of resource misallocation, such as the one underlying Hsieh and Klenow [2009], that for these distortions to have an effect on the economy beyond hindering allocative efficiency (e.g., by distorting entry, exit, and innovation choices), they must exhibit a fundamental relationship with the distribution of firms' physical productivity. Otherwise, distortions that create dispersion of marginal returns without a correlated component with productivity are neutral to the first moment of the firm size distribution.

Taking the distortion profile and the demand functions (defined in equations 4 and 2) as given, an intermediate good producer with productivity $(e^{\omega})^{\frac{1}{\theta-1}}$ chooses the optimal price and the optimal demand for labor by solv-

[11]It is easy to show that γ maps into the elasticity between $TFPR = \frac{1}{[1-\tau_w]}$ and $TFPQ = (e^{\omega})^{\frac{1}{\theta-1}}$ in the model.

ing the following static profit maximization problem[12]:

$$\pi^{v}(\omega) = max_{l(\omega)} (1-\tau_{\omega}) Y^{\frac{1}{\theta}} (e^{\omega})^{\frac{1}{\theta}} l(\omega)^{\frac{\theta-1}{\theta}} - wl(\omega),$$

which yields the familiar expression whereby labor demand, revenues, and profits are all proportional to idiosyncratic factors

$$l_t(\omega) = \left(\frac{\theta-1}{\theta}\right)^{\theta} \frac{Y_t}{w_t^{\theta}} e^{\omega} (1-\tau_{\omega})^{\theta} \tag{5}$$

$$\pi_t^{v}(\omega) = \frac{(\theta-1)^{\theta-1}}{\theta^{\theta}} \frac{Y_t}{w_t^{\theta-1}} e^{\omega} (1-\tau_{\omega})^{\theta}. \tag{6}$$

The factor of proportionality is determined by a mixture of aggregate variables to be solved in equilibrium, such as the wage rate (w) and final demand (Y), and the elasticity of substitution θ.

Firm Dynamics The tractability of the model emerges from simplifying the process of firm dynamics. We assume firms grow deterministically at rate μ upon entry and that they confront an exogenous probability δ of exiting the market. Later, we consider a richer setup with endogenous entry and endogenous innovation.

Normalizing the idiosyncratic productivity at birth to be equal to one (i.e., $e^{\omega(0)} = 1$), the exogenous growth rate μ implies that, conditional on survival, the idiosyncratic productivity of a firm of age a is given by

$$e^{\omega(a)} = e^{\mu a}.$$

As a result of the exogenous arrival of the exit shock δ, the fraction of firms in a cohort of entrants of measure M_e that remains alive at any given instant of time is $M_e e^{-\delta a}$. Expressing it as a fraction of the total number of firms in the economy, we obtain the following expression for the distribution of firms

[12]Unless essential for clarity, we are omitting the time subscript from static decision problems.

across ages:

$$f\left(a\right) = \delta e^{-\delta a},$$

where we are exploiting the notion that, in the stationary equilibrium, the number of firms is given by $M = \frac{M_e}{\delta}$.

The firm age distribution subsumes all of the heterogeneity in the economy and is thus sufficient to characterize an equilibrium. However, to draw an analogy with the richer quantitative model, we exploit the one-to-one mapping between age and productivity to characterize the equilibrium in terms of the productivity distribution of firms. Performing a change of variable, we obtain the following expression:

$$f\left(e^{\omega}\right) = \frac{\delta}{\mu}\left(e^{\omega}\right)^{-\left(1+\frac{\delta}{\mu}\right)}. \tag{7}$$

Notice that, as in Luttmer [2010], the resulting distribution is of the Pareto form, with the tail parameter given by the ratio of the exogenous death rate and the rate of productivity growth. The characterization of the *cross-sectional* distribution of firm productivity is sufficient for the aggregation of individual variables in the cross-section.

Consider now the prospects of profitability that accrue to potential entrants if they decide to confront the sunk costs of entry. Given the results from static optimization, the value of an entrant in units of labor is given by the lifetime expectation of profits:

$$\upsilon_e = \frac{\left(\theta-1\right)^{\theta-1}}{\theta^{\theta}\left(\rho+\delta\right)}\frac{Y}{w^{\theta}}\int e^{\mu a}\left(1-\tau_a\right)^{\theta}\left(\rho+\delta\right)e^{-\left(\delta+\rho\right)a}da.$$

Here ρ stands for the instantaneous discount factor of the household (to be defined below) and $\left(1-\tau_a\right)$ denotes the profile idiosyncratic distortion as a function of age. Notice that we have multiplied and divided by $\left(\rho+\delta\right)$ so that $\left(\rho+\delta\right)e^{-\left(\delta+\rho\right)a}$ can be interpreted as a probability density function of the distribution of productivity over age of an entering firm. We refer to this distribution as the *time-series* distribution of firms over ages, and as we did with the *cross-sectional* distribution, we can change variables and get a time-series distribution in the space of productivity:

$$f_{ts}\left(e^{\omega}\right)=\frac{(\delta+\rho)}{\mu}\left(e^{\omega}\right)^{-\left(1+\frac{\delta+\rho}{\mu}\right)}. \tag{8}$$

A notable property of the Pareto-shaped time-series distribution of productivity is the dependence of the tail parameter on the equilibrium interest rate in the economy, ρ. The role of the discount factor in the distribution is to reflect the differential valuation that potential entrants attach to profits at distant points in time. In probabilistic terms, discounting implies that the time-series distribution will attach a higher weight to the profits of young/unproductive firms compared to weight attached to young/unproductive firms by the distribution in the cross-section. This point of departure between distributions is essential for the results that follow, as it makes evident that productivity/age-dependent allocative distortions will have a differential valuation in the expectation of entrants' profits than in the average valuation of incumbents, which becomes decisive for determining the direction of change in firm entry and the average firm size in the economy.

Appealing to the productivity-dependent schedule of idiosyncratic distortions specified in equation 4, and exploiting the Pareto shape of the time-series distribution, we obtain the following expression for an entrant's value:

$$\upsilon_e=\frac{(\theta-1)^{\theta-1}}{\theta^{\theta}}\frac{Y}{w^{\theta}}\left[\frac{1}{\left(\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1\right)}\right]. \tag{9}$$

The entrant's expected profits are increasing in aggregate demand and productivity growth and are decreasing in wages, the elasticity between TFPR and TFPQ (γ), the probability of exit, and the interest rate.

Firm Entry Our theory of entry boils down to assuming that there is an infinite pool of potential entrants that, upon paying a labor-denominated entry cost f_e, earn the right to produce in the market and can start operations with a productivity level equal to $e^{\omega(0)}=1$ and growth rate μ conditional on survival.

In the distorted economy, we also assume that entrants must confront a tax rate τ^E per unit of entry cost so that, in equilibrium, the following free-entry condition must be satisfied:

$$f_e\left(1+\tau^E\right)=\upsilon_e.$$

Households Households are the ultimate owners of the portfolio of active firms in the economy. Their optimization problem consists of allocating their budget between consumption, savings in a risk-free bond $a\left(t\right)$, and investing in the form of firm creation. We abstract from population growth and assume a perfectly inelastic supply of labor.

Given an initial endowment of firms $M\left(0\right)$, the problem of the household is to maximize

$$\int e^{-\rho t}log\left[c\left(t\right)\right]dt,$$

subject to

$$\dot{a}\left(t\right)=r\left(t\right)a\left(t\right)+w\left(t\right)L+M\int\pi\left(a,t\right)f\left(a\right)da-f_eM_e\left(t\right)w\left(t\right)+T$$

$$\dot{M}=-\delta M+M_e, \tag{10}$$

where T stands for the transfer to/from the household that balances the budget from the collection of revenue from entry barriers and idiosyncratic distortions.[13]

The Euler equation ruling the incentives to accumulate assets implies that, in the stationary equilibrium, $\rho=r$.

Equilibrium and Aggregate Variables A competitive equilibrium in the model consists of (i) paths of consumption, firm entry, and asset accumulation $\left[c\left(t\right),a\left(t\right),Me\left(t\right),M\left(t\right)\right]_{t=0}^{\infty}$; (ii) paths of labor demands and prices for each

[13]The lump-sum transfer is how we implement the idea that we do not interpret τ^E and $[1-\tau_w]$ literally as taxes that drag resources away from the economy beyond the distortion they generate. In our model, output and consumption are reduced only to the extent that entry barriers and allocative distortions affect aggregate productivity.

variety $[l(\omega,t), p(\omega,t)]_{t=0}^{\infty}$; (iii) paths of demand for intermediate inputs and final output $[y(\omega,t), Y(t)]_{t=0}^{\infty}$; and (iv) a cross-sectional and time-series distribution of firms, a schedule of idiosyncratic distortions, and the entry barrier: $f_{ts}(e^{\omega})$, $f_{cs}(e^{\omega})$, $[1-\tau_{\omega}], \tau^{E}$. It also consists of (v) paths of interest rates and wages such that (ii) solves households' optimization problem subject to the budget constraint, the law of motion for the number of firms, and the paths of interest rates and wages; given wages, aggregate demand, and the profile of idiosyncratic distortions, (ii) solves each intermediate good producer's static profit maximization problem; given prices of varieties, (iii) solves the final good's static profit maximization problem; and the labor market clears, net asset demand is equal to zero, and the free-entry condition is satisfied:

$$L = M\left[\int l(w,t)\, f(e^{\omega})\, d(e^{w}) + \delta f_e\right] \tag{11}$$

$$f_e = \upsilon_e \left(1+\tau^{E}\right).$$

In what follows we focus on a *stationary equilibrium*, where the number of firms and consumption are constant. Thus, unless needed for conceptual clarity, we omit denoting time in the equilibrium functions.

Notice that in this simple model with no overhead costs and no continuing use of labor beyond production,[14] the average labor demand in production constitutes the empirically relevant measure of average firm size. Thus, denoting it with $\widehat{L_p}$, labor market clearing can be written as

$$L = M\left[\widehat{L_p} + \delta f_e\right]. \tag{12}$$

In terms of aggregate variables, it is straightforward to verify that in this model with CES aggregation of constant returns to scale production functions

[14] In the quantitative model developed later, firms also demand labor for fixed costs of operation and innovation activity.

and monopolistic competition, TFP is given by

$$TFP = M^{\frac{1}{\theta-1}} \frac{\left(\widetilde{\Omega}^{w}\right)^{\frac{\theta}{\theta-1}}}{\widetilde{\Omega}} * \frac{L_p}{L}, \tag{13}$$

where $\widetilde{\Omega}^{w}$and $\widetilde{\Omega}$ are statistics of the productivity distribution that capture the allocative efficiency in the economy.

Equation 13 lends itself for a decomposition of TFP into a static term capturing the allocative efficiency in the economy, $\frac{\left(\widetilde{\Omega}^{w}\right)^{\frac{\theta}{\theta-1}}}{\widetilde{\Omega}}$, and a dynamic term, the number of firms, capturing the preference for variety in demand. Idiosyncratic distortions (which we also refer to as allocative distortions) contribute to TFP both through the static and the dynamic component.[15] Entry barriers in turn are neutral to allocative efficiency in this model with exogenous exit, contributing to TFP only through the dynamic channel.[16] The share of production workers relative to the size of the labor force shows up in TFP because, as is the case in most national income and product accounts, expenses in firm creation are not capitalized, implying that the fraction of the labor force allocated to these activities makes no direct contribution to the economy's GDP.

3.1 Distortions, Number of Firms, and Average Size

This section characterizes the response of the average firm size and the number of firms to changes in the entry barrier and degree of misallocation. We are interested in assessing the direction of change in the number of firms and the average firm size in an economy with entry and allocative distortions relative to one where a single distortion is at play. Letting $M\left(\tau^{E}, \gamma\right)$ and $\widehat{L_p}\left(\tau^{E}, \gamma\right)$ be the number of firms and the average firm size in the economy with both entry

[15] A contribution of this section will be to characterize the direction of change in the number of firms as a function of the elasticity between the profile of idiosyncratic distortions and firm-level TFP.

[16] The quantitative model that we propose later features endogenous selection, thus opening up a channel for τ^{E} to also have an effect on allocative efficiency.

and allocative distortions, and letting $M\left(\tau^E,0\right)$ and $\widehat{L_p}\left(\tau^E,0\right)$ be the same objects when there are only entry barriers, we are interested in characterizing whether $\frac{M\left(\tau^E,\gamma\right)}{M(\tau^E,0)}$ and $\frac{\widehat{L_p}\left(\tau^E,\gamma\right)}{\widehat{L_p}(\tau^E,0)}$ are greater or lower than one.

Furthermore, to assess the interaction, we seek to establish the signs of $\frac{\partial\left(\frac{M\left(\tau^E,\gamma\right)}{M(\tau^E,0)}\right)}{\partial\tau^E}$ and $\frac{\partial\frac{\widehat{L_p}\left(\tau^E,\gamma\right)}{\widehat{L_p}(\tau^E,0)}}{\partial\tau^E}$. We proceed analogously when holding fixed the degree of misallocation, and we shut down the entry taxation. We present propositions and intuition in the main paper and defer proofs to Technical Appendix A.

3.1.1 Entry Barriers

Consider first the relationship between average firm size and the entry barrier τ^E. Taking the ratio of labor market clearing conditions between the economies with and without entry barriers and rearranging, we get

$$\frac{M\left(\tau^E,\gamma\right)}{M\left(0,\gamma\right)} = 1 + \left[\frac{\widehat{L_p}\left(0,\gamma\right)}{\widehat{L_p}\left(\tau^E,\gamma\right)} - 1\right]\omega_{L_p}, \tag{14}$$

where $\omega_{L_p} = \frac{\left(\widehat{L_p}\left(\tau^E,\gamma\right)\right)}{\left[\widehat{L_p}(\tau^E,\gamma)+\delta f_e\right]}$ denotes the share of production employment in total labor. By definition, $0 < \omega_{L_p} < 1$.

The equation establishes that the direction of change in the number of firms is inversely related to the change in the average size and the magnitude of the change is shaped by the share of production labor in total employment. Proposition 1 characterizes the change in the average firm size in response to an entry barrier.

Proposition 1. *Let the death rate and the productivity growth rate of firms be such that* $\frac{\delta}{\mu} > 1$, *let misallocation be characterized by equation 4 with* $0 < \frac{\theta}{\theta-1}\gamma < 1$, *and let the entry barrier be given by* τ^E; *then the average size of firms in the distorted economy with* $\left\{\tau^E > 0, \gamma \geq 0\right\}$ *relative to an economy* *with* $\left\{\tau^E = 0, \gamma \geq 0\right\}$ *is given by*

$$\frac{\widehat{L_p}\left(\tau^E, \gamma\right)}{\widehat{L_p}\left(0, \gamma\right)} = \left(1 + \tau^E\right) > 1.$$

Proposition 1 formalizes the result that entry barriers increase the average firm size relative to an equilibrium with no distortions to entry, independently of the underlying degree of misallocation. As a byproduct, it follows that the number of firms falls in response to the imposition of an entry barrier, as can be readily seen by substituting the ratio of average size from proposition 1 into equation 14:

$$\frac{M\left(\tau^E, \gamma\right)}{M\left(0, \gamma\right)} = 1 - \frac{\tau^E}{1 + \tau^E} * \omega_{L_p} < 1.$$

Since, with CES aggregate technologies and exogenous exit, the number of firms is the sole channel through which entry shapes TFP, the proposition is in fact showing the expected result that entry barriers reduce aggregate productivity through a reduction in the number of varieties.

In terms of the mechanisms behind the results, the rise in average size is driven by the general equilibrium response of the aggregate demand and the wage rate. At the given value of these two, the entry barrier is neutral to the expected profitability of entrants. However, it makes entry more costly, breaking the balance in the free-entry condition, reducing aggregate demand of labor for entry purposes, and generating excess supply in the labor market. As a result, $\frac{Y}{w^{\theta}}$ starts to increase until both equilibrium conditions are satisfied. In the new equilibrium, the more favorable aggregate conditions increase the average demand for production labor and reduce the number of firms.

3.1.2 Idiosyncratic Distortions

Consider now the sensitivity of equilibrium variables to changes in the degree of misallocation, as captured by changes in the elasticity of the TFPR − TFPQ profile in equation 4. The ratio of the number of firms between the economy with both entry and allocative distortions and the one with entry barriers only is given by

$$\frac{M\left(\tau^{E},\gamma\right)}{M\left(\tau^{E},0\right)}=1+\left[\frac{\widehat{L_p}\left(\tau^{E},0\right)}{\widehat{L_p}\left(\tau^{E},\gamma\right)}-1\right]\omega_{L_p}, \tag{15}$$

where, again, ω_{L_p}denotes the production employment share in the economy with multiple distortions, $\left\{\tau^{E},\gamma\right\}$. As before, the direction of change in the number of firms is intricately related to the response in average size, while the magnitude of the change is also shaped by the production employment share. Proposition 2 characterizes the response in average size.

Proposition 2. *Let the death rate and the productivity growth rate of firms be such that* $\frac{\delta}{\mu}>1$, *let misallocation be characterized by equation 4 with* $0<\frac{\theta}{\theta-1}\gamma<1$, *and let the entry barrier be given by* τ^{E}. *Then average firm size in the economy with distortions* $\left\{\tau^{E},\gamma\right\}$ *relative to* $\left\{\tau^{E},0\right\}$ *is given by*

$$\frac{\widehat{L_p}\left(\tau^{E},\gamma\right)}{\widehat{L_p}\left(\tau^{E},0\right)}=\frac{\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]}{\left[\frac{\frac{\delta+\rho}{\mu}-1}{\frac{\delta}{\mu}-1}\right]}<1\Longleftrightarrow\rho>0. \tag{16}$$

Proposition 2 confirms[17] the result that the average size of firms is lower in economies with misallocation profiles that feature a steeper elasticity between TFPR and TFPQ, provided the interest rate in the economy is positive. Furthermore, the change in average firm size in response to the misallocation elasticity is independent from the entry tax rate τ^{E}, dictating the absence of an interaction between distortions when it comes to average firm size. Last, substituting the change in average size back into the number of firms in equation (15), we see that the decline in average size is sufficient to imply an increase in the number of firms if and only if $\rho>0$:

$$\frac{M\left(\tau^{E},\gamma\right)}{M\left(\tau^{E},0\right)}=1+\frac{\frac{\theta}{\theta-1}\gamma\frac{\rho}{\mu}}{\left[\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1\right]\left[\frac{\delta}{\mu}-1\right]}\omega_{L_p}.$$

[17] Bento and Restuccia [2017], for instance, appeal to this result to rationalize cross-country differences in average firm size with cross-country differences in the degree of productivity-dependent idiosyncratic distortions.

Unlike the case of entry barriers, the equilibrium response of the number of firms contributes to mitigating the TFP loss from misallocation in the long run, as it provides a rise in the number of varieties that partly offsets the efficiency losses from misallocation.[18]

Turning to the intuitions, the simple model helps clarify the mechanisms through which the above results emerge, highlighting the key roles played by the interest rate and the productivity dependence of misallocation. The interest rate (ρ) matters because it determines the extent to which entrants discount future profit flows. When the elasticity of misallocation (γ) goes up, it induces a redistribution of profits from more productive to less productive firms and, given the growth process of productivity upon entry, a reallocation of profits from the future to the present. Such redistribution is detrimental to both average and expected productivity. However, due to discounting, it is less detrimental to the time series than to the cross-sectional average. Therefore, the adjustment in aggregate demand and wages that needs to take place to restore the free-entry condition does not fully undo the decline in average labor demand from incumbents, ultimately giving rise to a decline in the average size and an increase in firm entry.

3.1.3 Interaction between Distortions

Consider now the possibility of an interaction between distortions. We saw in propositions 1 and 2 that the response in average size is independent of the distortion that is not changing. However, there is scope for distortions to interact in shaping the change in the number of firms since this change is also determined by the production employment share in the distorted allocation, which is sensitive to the magnitude of the underlying friction. The following proposition characterizes the interaction's direction.

Proposition 3. *Let the death rate and the productivity growth rate of firms be such that* $\frac{\delta}{\mu} > 1$, *let misallocation be characterized by equation 4 with* $0 <$

[18] Fattal-Jaef, 2018 shows that while protective of TFP in the long run, the rise in the number of firms magnifies the overall welfare losses once the transitional costs of accumulating more firms are taken into account.

$\frac{\theta}{\theta-1}\gamma < 1$, *and let the entry barrier be given by* τ^E*. Then the change in* $\frac{M(\tau^E,\gamma)}{M(\tau^E,0)}$ *with respect to changes in* γ*, and the change in* $\frac{M(\tau^E,\gamma)}{M(\tau^E,0)}$ *with respect to changes in* τ^E*, are given by*

$$\frac{\partial\left[\frac{M(\tau^E,\gamma)}{M(0,\gamma)}\right]}{\partial\gamma} = \frac{-\tau^E}{(1+\tau^E)} * \frac{\partial\omega_{L_p}}{\partial\gamma} > 0$$

$$\frac{\partial\left[\frac{M(\tau^E,\gamma)}{M(\tau^E,0)}\right]}{\partial\tau^E} = \frac{\frac{\theta}{\theta-1}\gamma\frac{\rho}{\mu}}{\left[\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1\right]\left[\frac{\delta}{\mu}-1\right]} * \frac{\partial\omega_{L_p}}{\partial\tau^E} > 0,$$

where

$$\frac{\partial\left(\omega_{L_p}\right)}{\partial\gamma} = \delta * \frac{\left[\left(1+\tau^E\right)(\theta-1)\frac{\delta}{\mu}\frac{\theta}{\theta-1}\left(\frac{-\frac{\rho}{\mu}}{\left(\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1\right)^2}\right)\right]}{\left\{\left(1+\tau^E\right)(\theta-1)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]+\delta\right\}^2} < 0$$

$$\frac{\partial\omega_{L_p}}{\partial\tau^E} = \frac{\frac{\delta}{\mu}(\theta-1)\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]\delta}{\left[\left(1+\tau^E\right)(\theta-1)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]+\delta\right]^2} > 0.$$

Proposition 3 establishes that entry and allocative distortions mitigate each other's detrimental effect on TFP.[19] More stringent misallocation, manifested in a higher elasticity between TFPR and TFPQ, weakens the decline in the number of firms induced by the introduction of an entry barrier, whereas higher entry taxes magnify the increase in the number of firms resulting from correlated allocative distortions.

Consider first the intuition for why entry barriers magnify the rise in the number of firms and therefore minimize the TFP losses from idiosyncratic distortions. As seen clearly in equation 15, idiosyncratic distortions induce a

[19] Recall that we are characterizing the long-run implications of distortions. Therefore, the fact that each distortion mitigates each other's effect on long-run TFP does not necessarily mean that the same is true about welfare, for which transitional dynamics should also be accounted for.

reduction in the average firm size, which frees up resources to be allocated to firm creation. When this type of distortion is implemented in a context of high entry barriers, the starting average size is high, so the forces that induce the reallocation of workers apply to a higher base.

Similarly, the intuition for why idiosyncratic distortions mitigate the fall in the number of firms induced by entry barriers is also connected to how these distortions affect the base over which the general equilibrium forces of the entry barrier apply. As seen in equation 14, the number of firms falls because entry barriers increase average firm size, which absorbs resources away from firm creation. When these barriers are implemented in a context of severe misallocation, the starting average size is lower, so the given percentage change in average size induced by the entry barriers apply to a lower base.

3.1.4 Discussion

In this section we have shown that the average firm size subsumes information about the relative weight between entry barriers and idiosyncratic distortions. Learning about their absolute values, however, is inhibited by the existence of a single equilibrium condition in the model to identify the two unknowns. Since Hsieh and Klenow [2009]'s approach to measuring idiosyncratic distortions does not require knowing the equilibrium prices and aggregates,[20] one can get an estimate of the distortion slope γ directly from the data and use the model's equilibrium condition to back out the entry barrier that matches the equilibrium's average firm size with the average firm size in the data. We implement this strategy later in the paper.

Notice from the logic above that the identified entry tax rate is sensitive to three potential sources of bias: 1) measurement error in average size, 2) measurement error in the elasticity of TFPR $-$ TFPQ, and 3) model misspecification. The next section presents a series of adjustments to our firm-level

[20]Recall from Hsieh and Klenow [2009] that wedges are defined as deviations of marginal revenue products relative to industry averages. Thus, industry-wide variables related to the equilibrium, such as aggregate demand and wages, are subsumed in this average of revenue products across firms within an industry.

databases that attenuate biases stemming from the first two sources. Furthermore, to minimize model misspecification, we conduct the quantitative analysis by appealing to a richer version of the model just presented that incorporates two key determinants of the size distribution: endogenous exit and endogenous firm dynamics.

Last, the theoretical results concerning the interaction between distortions will prove helpful when interpreting the magnitude of the TFP effects associated with altering the mix of distortions.

4 Data

The identification of entry barriers hinges on two empirical inputs: the average firm size and the elasticity of idiosyncratic distortions with respect to firms' physical productivity (i.e., the TFPR and TFPQ elasticity). For the purpose of extracting these inputs, we gathered a collection of firm-level databases from 21 countries, to which we have made adjustments so as to maximize the comprehensiveness and the comparability of the sample.

The data are from a combination of manufacturing censuses of firms, collected by each country's statistical agencies, and the Amadeus database, compiled by the Bureau Van Dijk, which collects the financial information of public and private firms from 34 European countries. The manufacturing censuses correspond to Ghana, Ethiopia, Kenya, Chile, Colombia, Peru, India, Malaysia, Bangladesh, Pakistan, and El Salvador. In the majority of the cases, the censuses are meant to be universal for firms of a certain size, typically ten workers, with the exceptions of Peru and El Salvador, for which there are no size constraints. The Amadeus database in turn does not impose size restrictions, but it is well known that for some countries there is a bias toward over-representing the largest firms. Given these differences, we apply some adjustments to maximize comparability and to minimize biases in the representation of the size distributions.

The first adjustment we are forced to implement is restricting the analysis to firms with 10 workers or more. The benefit of this choice is that it maximizes comparability across all of the data sources. Of course, the downside is the lack of information on the prevalence and behavior of the smallest firms in the size distribution. Our assumption is further supported by observing that existing cross-country estimates of the costs of doing business are in effect based on the legal costs of creating a 10-plus worker firm. For example, the World Bank's DBI (a leading source of information for the calibration of entry barriers in the literature[21]) computes the cost of starting a business as the summation of all the processes entrepreneurs must undergo to obtain the necessary approvals, licenses, and permits for operating a new firm. The prototypical firm for the measurement of these costs is precisely one that features 10 to 50 employees one month after operations start. Thus, we see our analysis as providing a characterization of the business environment that rationalizes the cross-country differences in the average size distribution of firms with 10 workers or more. Of course, the truncation to the data requires that the model is calibrated to match properties of the truncate firm size distribution in the US, our target economy, a requirement we take into account when calibrating our model.

The second adjustment concerns the selection of countries from the Amadeus database. Given the known heterogeneity in the degree of coverage of the countries' firm size distributions, and the importance of minimizing biases in the calculation of average firm sizes, we apply the following selection criteria. Taking Eurostat's Structural Business Statistics database as the measure of the true size distribution of employment, we keep a country in the sample if 1) the ratio of aggregate manufacturing employment in Amadeus to aggregate manufacturing employment in Eurostat is greater than or equal to 80 percent, and 2) the ratio of manufacturing employment between Amadeus and Eurostat in each bin of Eurostat's size distribution above the 10-worker threshold (10–20, 20–50, 50–250, 250+) is also greater than or equal to 80 percent. The countries that satisfy these criteria are Belgium, Bulgaria, Finland, France, Hungary, Italy, Latvia, Portugal, Romania, and Spain. Table 2 in Data Appendix B shows the final list of countries together with a brief description of

[21]See, for instance, Barseghyan and DiCecio [2011] and Boedo and Mukoyama [2012].

the data sources.

4.1 Average Size and Idiosyncratic Distortions

One advantage of using the firm-level data to compute average size is that it allows us to control for cross-country differences in production structures across broad industries within manufacturing. Thinking of manufacturing as a collection of two-digit sub-industries, average size may then vary across countries due to 1) heterogeneity in average size across firms within two-digit industries under a fixed distribution of production shares across these industries and 2) heterogeneity in production shares for a given distribution of average sizes. Considering this distinction is essential for our analysis given our interpretation of cross-country differences in average firm size as being driven by differences in the degree of allocative distortions and entry barriers. Since, following the literature (Hsieh and Klenow [2009]), misallocation is measured across firms within an industry,[22] our theory is silent about potential misallocation across manufacturing industries. Thus, the measure of average size that our theory can speak to is one that abstracts from the international variation in production structures.

Our strategy to control for cross-country differences in production structures entails fixing the distribution of firms across two-digit manufacturing industries to be given by the distribution in the US.[23][24] More specifically, letting $AvSize_i^{FD}$ stand for the average firm size in country i under a fixed

[22]The reason to restrict the measurement of misallocation to firms within narrow industries is that this is the context in which the underlying assumption of common production technologies and common factor prices across firms is more likely to apply. Put differently, it is at the level of a narrow industry that it is more reasonable to expect that firms should behave identically across countries. At a more aggregate level, other factors (comparative advantage, geography) make it more plausible countries exhibit a different production structure and that such difference is an efficient outcome.

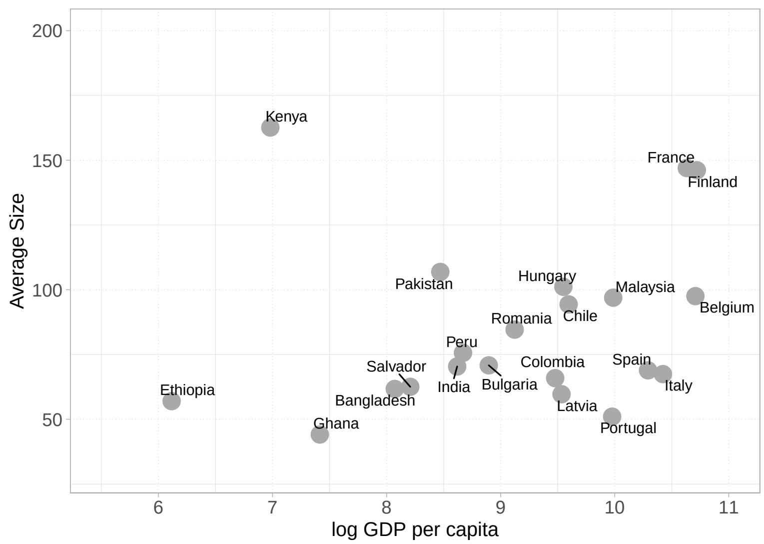
[23]Appendix B.1 shows that controlling for industrial composition reduces the average firm size in many countries, relative to what would have resulting from the adoption of each country's distribution of firms across 2-digit industries. In the rest of the countries, the impact on average firm size is minimal.

[24]We read the distribution of firm shares across two-digit industries in the US' manufacturing sector from the Small Business Administration database for 2007.

distribution of firms, and letting $\left(\frac{M_s}{M}\right)^{US}$ denote the share of firms in any given two-digit sector s in the US, then

$$AvSize_i^{FD} = \Sigma_{s=1}^{S} AvSize_s * \left(\frac{M_s}{M}\right)^{US}. \tag{17}$$

Figure 1: Average Size and Development



Note: The vertical axis measures the average firm size, conditional on 10-plus workers, applying the US' distribution of firms across two-digit industries to all countries. The horizontal axis measures the natural logarithm of GDP per capita, for the year, 2014, as reported in the Penn World Table version 9.0 (Feenstra et al. 2015, Zeileis 2019). The list of countries in the sample is reported in Table 2 of Appendix B.

Appendix C provides theoretical validation for our aggregation strategy in the context of a two-sector extension of the model, where comparative advantage is driven by exogenous sectoral differences in TFP.

Equipped with our preferred definition, we revisit the relationship between firm size and economic development. Figure 1 confirms the positive relationship between firm size and income per capita documented earlier in the literature,[25] showing a correlation between the log of variables of 0.28.

We turn now to estimating the elasticity between idiosyncratic distortions (TFPR) and idiosyncratic productivity (TFPQ). As in Hsieh and Klenow

[25] Salient examples of recent work documenting the relationship between average firm size and economic development in the data are Bento and Restuccia [2017] and Poschke [2014].

[2009], misallocation in our model manifests as a dispersion in the marginal revenue products of firms within an industry. The property of the resource misallocation that matters for the purpose of understanding changes in average firm size is given by the elasticity between TFPR and TFPQ. We estimate this elasticity by running the following regression for each country in the sample[26]:

$$log\left(\frac{TFPR_i}{\overline{TFPR}_s}\right) = \alpha + \gamma log\left(\frac{TFPQ_i}{\overline{TFPQ}_s}\right), \tag{18}$$

where $\overline{TFPR}_s$ and $\overline{TFPQ}_s$ stand for the average revenue and physical productivity in industry s.

Unlike previous estimates of the distortion elasticity, such as Bento and Restuccia [2017] and Buera and Fattal-Jaef [2018], we pursue an employment weighted least squares (WLS) estimation. The goal of the weighting is to consider the possibility that the most productive firms, which account for a larger share of employment, may be less subject to idiosyncratic distortions, particularly so in advanced countries. Indeed, Appendix B.2 shows that ordinary least squares (OLS) yields very similar estimates to WLS in the least developed countries, whereas it predicts substantially more correlated misallocation in advanced economies, a feature that resonates as implausible.[27]

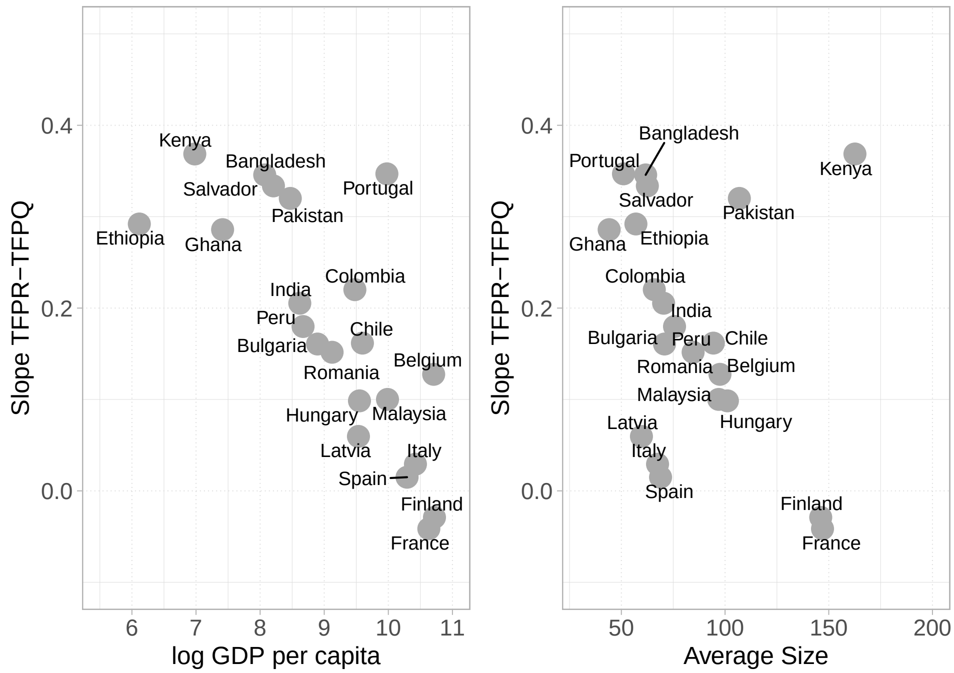
The left panel of Figure 2 shows, as expected, a negative relationship between the productivity elasticity of distortions and GDP per capita. In particular, advanced economies such as Italy, Spain, France, and Finland feature an elasticity virtually on par with the estimate for the US, a feature that is reassuring of the WLS estimate's accuracy. Conversely, distortions are most strongly correlated with productivity in the poorest economies, such as Kenya, Ghana, and El Salvador.

The right panel of Figure 2 turns to the relationship between the produc-

[26]Given that the US, our efficient benchmark, also exhibits correlated distortions, we subtract the US elasticity from the estimated one for each country. We read the US distortion elasticity from Hsieh and Klenow [2007], which reports a value of 0.18 for the year 1998.

[27]For instance, under OLS, Belgium would show a higher TFPR – TFPQ elasticity than India and would be on par with Chile, whereas France would be portrayed as misallocated as India.

Figure 2: TFPR – TFPQ Elasticity, GDP, and Average Size



Note: The left panel of figure illustrates the weighted least squared (WLS) estimate of the regression defined in equation 18, with firm-level employment shares as weights, against the log of income per capita for the year 2014, as reported in the Penn World Table version 9.0. A regression coefficient of 0.15 for the US was subtracted from each country's estimate to control for misallocation in the efficient benchmark. The measurement of TFPR preserves the parametric assumptions of Hsieh and Klenow [2009], namely an elasticity of substitution equal to 3, and factor shares taken from the US four-digit manufacturing sector. The tails of the TFPR and the TFPQ distribution were trimmed at the bottom and top 5 percent. The right panel projects the TFPR – TFPQ elasticity against average firm size constructed according to equation 17

tivity elasticity of distortions and average firm size in the data. As predicted by the model, the figure shows a negative relationship between the productivity elasticity of distortions and the average firm size. Considered together, both panels of Figure 2 reinforce the plausibility of idiosyncratic distortions as drivers of cross-country differences in productivity. These can account not only for aggregate differences in performance but also for micro differences in average firm size. The point we develop next, however, is that when interpreted through the lens of a rich model of firm dynamics, the average firm size responds too strongly to the idiosyncratic distortions, which calls for a countervailing force to account for the average size in the data.

5 Quantitative Analysis

In this section we conduct the inference of entry barriers and the quantification of the macroeconomic consequences of each distortion. First, we extend the simple model to incorporate features of the economy that are central to its responsiveness to distortions. These comprise the endogenous exit and the endogenous innovation decisions of firms. Second, we present the strategy for the calibration of parameter values. Then, based on average size and misallocation estimates from Section 4.1, we continue with the identification of entry taxes to finally compute the potential TFP gains associated with the partial and complete reversal of these distortions.

5.1 Quantitative Model

The quantitative model enriches the current setup by introducing fixed costs of production and considering endogenous firm dynamics. Firms incur a labor-denominated fixed costs of operation f_c to remain in operation. As a result, firms whose profitability falls below a certain threshold will be forced out of the market. Entry barriers and idiosyncratic distortions will have an effect on the stringency of this threshold.

Endogenous firm dynamics arise from a process of technological upgrading and downgrading similar to that in Atkeson and Burstein [2010]. Specifically, a firm with current productivity e^{ω} can upgrade to $e^{\omega+\Delta}$ with probability $q_t(\omega)$ and can downgrade to $e^{\omega-\Delta}$ with probability $(1-q_t(\omega))$. The expected growth rate, given by $q_t(\omega)$, is endogenous, as firms can allocate resources to innovation activities. The variance of the shock process (Δ), on the other hand, is exogenous. The labor-denominated cost for attaining a desired probability $q_t(\omega)$ is given by

$$\chi(q_t, \omega) = e^{\omega} \times \eta\left(e^{\phi q_t} - 1\right).$$

Notice that the innovation cost is scaled by the entrepreneur's current productivity. This is an important assumption that allows the model to be consistent with innovation patterns of large firms in the US, our target economy for the calibration. The scale parameter η and the elasticity parameter ϕ will be calibrated to replicate the properties of the size distribution and the life cycle of firms in the US.[28]

The value of an operating incumbent firm with current productivity e^{ω} is

$$v_t^o(\omega) = \max_{q_t(\omega)} \left\{ \begin{array}{c} \pi_t^v(\omega) - w_t \chi(q_t, \omega) - w_t f_c \\ +R_t(1-\delta)\left[q_t(\omega) v_{t+1}\left(e^{\omega+\Delta}\right) + (1-q_t(\omega)) v_{t+1}\left(e^{\omega-\Delta}\right)\right] \end{array} \right\} \tag{19}$$

where $\pi_t^v(\omega)$ is the indirect variable profit function under optimal factor demands given prices and distortions, as defined in equation 6, and f_c denotes the labor-denominated fixed cost of production.

The value of the firm is given by

$$v_t(\omega) = max_{\iota_t(\omega)} \left\{v_t^o(\omega), 0\right\},$$

where $\iota_t(\omega)$ encodes the firm's exit decision, equal to one if it operates and equal to zero if it exits.

The first-order condition with respect to the innovation choice yields

$$w_t \phi \mu e^{\phi p(\omega)} = R_t(1-\delta)\left[v_{t+1}\left(e^{\omega+\Delta}\right) - v_{t+1}\left(e^{\omega-\Delta}\right)\right].$$

Firms choose the probability of a technological upgrade so as to equate the marginal cost of innovation efforts with the difference in firm valuation. To the extent that idiosyncratic distortions are productivity dependent, as we are assuming, these distortions will have a direct contribution to the rate of return to innovation expenses, in addition to general equilibrium effects on wages and final demand. Notice that entry barriers, which we have not introduced yet, have no direct effect on the firms' innovation decisions (though they will still have an indirect contribution through general equilibrium forces).

Firm entry emerges as in the simple model, except in this version we assume ex-post heterogeneity upon entry. In other words, there is an infinite pool of

[28] The process for idiosyncratic productivity can be interpreted as a binomial approximation to a geometric Brownian motion, with an exogenous standard deviation Δ and endogenous drift $(2p_t(z) - 1)\Delta$.

potential entrants that, upon paying a labor-denominated sunk entry cost f_e, draw a level of idiosyncratic productivity from the known distribution $\Gamma(\omega)$ and the associated idiosyncratic distortion $\tau(\omega)$ and decide whether to become an active business or exit the market immediately. The free-entry condition in this context is given by

$$w_t f_e (1 + \tau^e) = R_t (1 - \delta) \int v_{t+1}(\omega) \, d\Gamma(\omega) . \tag{20}$$

Notice that we are assuming a one-period time to build between the payment of the entry cost and the actual entry.

The definition of an equilibrium preserves the structure of the simple model, the sole difference being that labor market clearing now reflects the use of labor for innovation and fixed costs of operation, which we denote with $L_{fc,t}$ and $L_{I,t}$ and are defined by

$$L_{fc,t} = f_c \int dM_t(\omega)$$

$$L_{I,t} = \int \chi(q_t, \omega) \, dM_t(\omega) .$$

The labor market clearing condition then becomes

$$L = L_{p,t} + L_{fc,t} + L_{I,t} + f_e M_{e,t}.$$

Last, the law of motion for the distribution of firms across productivity levels must also reflect the new structure of the stochastic process and the possibility of firm exit. Formally, letting $M_{t+1}(\omega')$ be the mass of firms with a log-productivity level less than or equal to ω, the productivity distribution of firms evolves according to the following law of motion:

$$M_{t+1}(\omega') = (1 - \delta) q_t (\omega' - \Delta) M_t(\omega' - \Delta) \tag{21}$$

$$+ (1 - \delta) [1 - q_t (\omega' + \Delta)] M_t(\omega' + \Delta) + (1 - \delta) M_{e,t} \Gamma(\omega') .$$

The expression establishes that a fraction $(1 - \delta) q_t (\omega' - \Delta)$ of firms with productivity less than or equal to $(\omega' - \Delta)$ survives the exogenous exit shock and transitions to a productivity level less than or equal to ω'. A fraction $(1-\delta)\left[1-q_t\left(\omega'+\Delta\right)\right]$ of the mass of firms with productivity between ω' and $\left(\omega'+\Delta\right)$ survives the exit shock and jumps downward to have productivity less than or equal to ω'. There is also an inflow of new firms into this group, which is given by the mass of entrants, a fraction $\Gamma\left(\omega'\right)$ of which will feature a productivity less than or equal to ω'. Endogenous exit will be driven by the mass of firms that transition downwards from the productivity cutoff, $(1-\delta)\, q_t\left(\underline{\omega}+\Delta\right) M_t\left(\underline{\omega}+\Delta\right)$.

The introduction of new sources of demand for labor requires that we take a stand on the empirically consistent definition of aggregate labor demand that we should use to define average size in the model. Following the practice of our various firm-level databases, which define total employment of the firm as the aggregate across all of its workers, ranging from production to managerial and research positions, we measure firm size as the aggregate of employment across production $\left(l\left(\omega\right)+f_c\right)$ and innovation $\left(\chi\left(q_t,\omega\right)\right)$ purposes. Formally, then, average firm size in the model is given by

$$AvSize_t=\frac{\int\left[l_t\left(\omega\right)+fc+\chi\left(q_t,\omega\right)\right]dM_t\left(\omega\right)}{\int dM_t\left(\omega\right)}. \tag{22}$$

Since, as we explain below, most of our firm-level databases only cover firms with ten or more workers, the model-based counterpart of the average size should is adjusted accordingly. TFP is still defined as in equation 13.

5.2 Calibration

We must choose parameter values for the elasticity of substitution θ, the subjective discount factor of the household β, and the set of parameters governing the process of firm dynamics, entry and exit: entry and fixed operation costs f_e and f_c, the size of the jump in the binomial process Δ, the exogenous exit rate δ, and the parameters in the innovation cost function, η and ϕ. For calibration purposes, we consider the economy with no distortions and target moments of the firm size distribution and the life cycle of firms in the US.

Table 1 summarizes the parameter values and the matching moments.

Table 1: Parameter Values and Calibration Targets

Parameter	Value	Target
ρ	3	Hsieh and Klenow (2009), Broda and Weinstein (2006)
β	$\frac{1}{1.05}$	Interest rate of 5%
δ	0.025	Employment-based exit rate of large firms of 2.5%
η	0.00056	Top 10% employment share = 0.77
ϕ	15	Employment age 21–25 relative to age 1
Δ	0.25	Std dev. of employment growth of large firms
$G(\omega)$	$\omega_0 = 0$	Size of entrants = 20% of average incumbent
$\frac{f_c}{f_e}$	0.1	Exit rate of 8.6%
f_c	3.165	US average firm size conditional on $L \geq 10 = 118$

The top 10 percent employment share, the average employment ratio between 21–25 and 1-year-old firms, the average employment ratio between entrants and incumbents, and the average firm size conditional on $L \geq 10$ were computed from the Business Dynamics Statistics database for the year 2007. Numbers are for the manufacturing sector. Standard deviation of employment growth rates for large firms are reported in Atkeson and Burstein [2010].

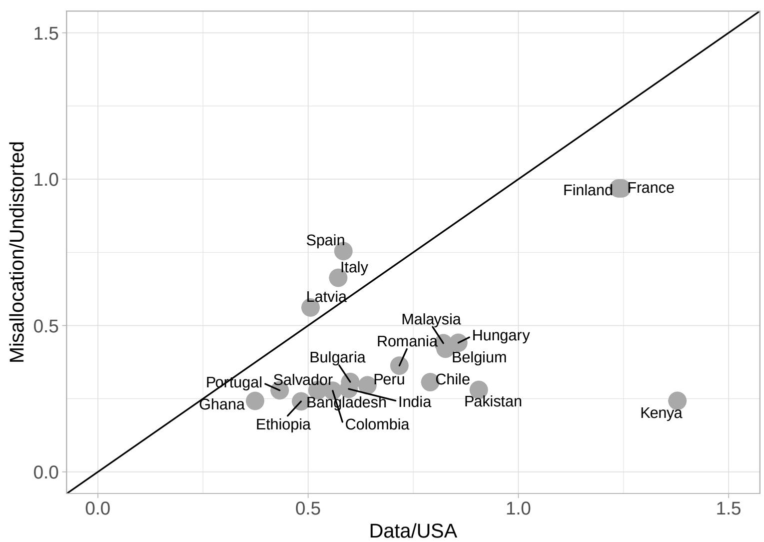
While the calibration strategy is standard, it is important to highlight the calibration of the fixed cost of production, which is set to target the average firm size in the US manufacturing sector among firms with ten or more workers. As explained in the data section, our cross-country firm-level database restricts us to studying forms above the ten-worker threshold, so we adjust the average size in our undistorted model economy to attain the average size of the benchmark economy, the US, when constrained to the same universe of firms.

5.3 Cross-Country Distribution of Average Firm Size under Idiosyncratic Distortions Only

As motivation for the importance of entry barriers, we begin characterizing the model's prediction for the cross-country distribution of average firm size under the assumption that idiosyncratic distortions are the sole distortion in the economy. As established in proposition 2, idiosyncratic distortions reduce the average firm size in equilibrium, thus constituting a plausible driving force for the cross-country distribution of average size in the data. However, our goal here is to show that idiosyncratic distortions fail to account for such distribution, thus reenacting the relevance of entry barriers.

Formally, the experiment consists of feeding each country's estimate of the productivity elasticity of distortions (γ_i) into the quantitative model previously presented, solving for the stationary equilibrium, and comparing the resulting average size with the data. The results from this exercise are reported in Figure 3. The plot in the figure illustrates the average firm size from the model's distorted stationary equilibrium relative to the undistorted one, against the average firm size ratio each country with respect to the US. Shortcomings in the model are reflected as deviations from the 45-degree line.

Figure 3: Average Firm Size across Countries under Idiosyncratic Distortions Only



Note: The figure illustrates the average size in the model with idiosyncratic distortions only relative to the undistorted one, against the ratio of the average size in each country relative to the US. Each country's stationary equilibrium is solved under the distortion elasticity reported in Figure 2.

Figure 3 illustrates the (in)ability of the economy with idiosyncratic distortions to replicate the average firm size distribution in the data. With few exceptions, we find that the decline in average firm size implied by the economy with idiosyncratic distortions is notably larger than observed in the data. For the majority of countries, then, reconciling the equilibrium's average firm size with the empirical one requires a countervailing force on the firm size distribution. As an example, consider the case of Chile. In the data, Chilean manufacturing firms are 80 percent the size of US manufacturing firms. Furthermore, the Chilean economy features a productivity elasticity of distortions of 0.17 (recall this is net of the US degree of correlated distortions). According to the model, had the idiosyncratic distortions been the sole friction, the average firm size in Chile would be 30 percent the size in the US. Increasing the average size to the observed 80 percent ratio then requires a complementary distortion that counteracts the correlated misallocation. We argue that this countervailing force reflects entry barriers in the economy.

5.4 Model-Based Estimates of Entry Barriers

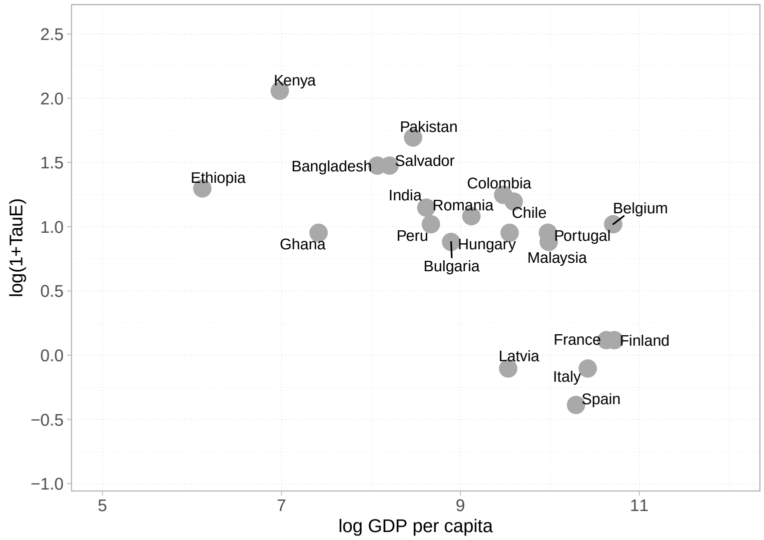
Convinced about the need of a complementary force to account for the cross-country distribution of average firm size, this section implements a strategy to identify such force as a barrier to firm entry. Since these barriers increase the average firm size for any given value of the distortion productivity elasticity, the strategy involves iterating on the space of τ_i^E, taking γ_i as given, until the distorted equilibrium's average firm size coincides with the empirical one (i.e., until the dots in Figure 3 are all lined up in the 45-degree line).

Formally, given the estimate of γ_i, we guess a value of τ_i^E and solve for the distorted competitive equilibrium under distortion pair $\left\{\tau_i^E, \gamma_i\right\}$. We compute the equilibrium average size ratio with respect to the undistorted economy and compare it against the average size in the data relative to the US. Guided by the theoretical results, we update the guess of τ_i^E upwards if the average size still falls below the data and update it downwards if it rises above. [29] The results are plotted in Figure 4.

The figure shows that entry barriers are sizable in the least developed economies and decrease to roughly zero in the most advanced ones. The model backs out mild entry subsidies for Latvia, Spain, and Italy, which are the

[29] Notice from Figure 3 that countries for which the average size under idiosyncratic distortions only lie above the 45-degree line (Latvia, Spain, Italy), the model will back out a negative value of the entry barrier (i.e., an entry subsidy).

Figure 4: Model-Based Entry Tax Rate versus GDP per Capita



Note: The figure plots the model's estimate for the entry tax τ^E against the log of GDP per capita. Data for GDP per capita correspond to the year 2014 and are drawn from the Penn World Table version 9.0 (Feenstra et al. 2015, Zeileis 2019).

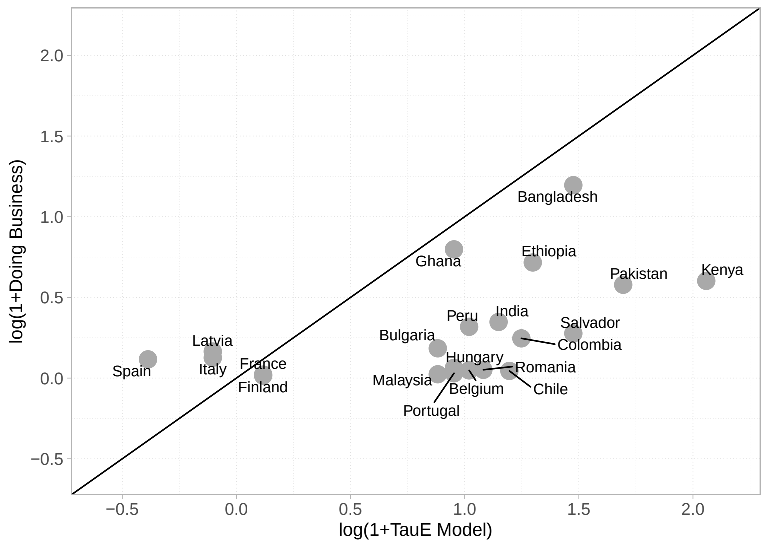
countries that lie above the 45-degree line in the left panel of Figure 3.

As a first attempt to give meaning to the magnitudes, Figure 5 plots the model's estimate of entry barriers against the World Bank's DBI for the cost of starting a firm. This indicator, which is based on legislation and regulations, has been used widely in the literature to quantify the aggregate effects of barriers to firm entry. While the DBI comprises various elements, we focus on those that more directly affect the costs of starting a formal manufacturing firm: the actual time cost of getting permits and licenses to operate a new business plus the costs of getting access to electricity. The DBI translates these costs into fractions of GDP per capita, which we then turn into labor units to make them comparable with $f_e * \tau^E$ in the model.[30]

Figure 5 provides information on the degree of correlation and the relative magnitude between the two estimates of entry barriers. We find that model-based estimates correlate strongly with the DBI's cost of starting a firm, with a correlation of 0.51. The comparison of magnitudes in turn reveals that the

[30]See the note to Figure 5 for a description of this conversion.

Figure 5: Model-Based Estimate of Entry Barriers and the World Bank's Doing Business Indicators

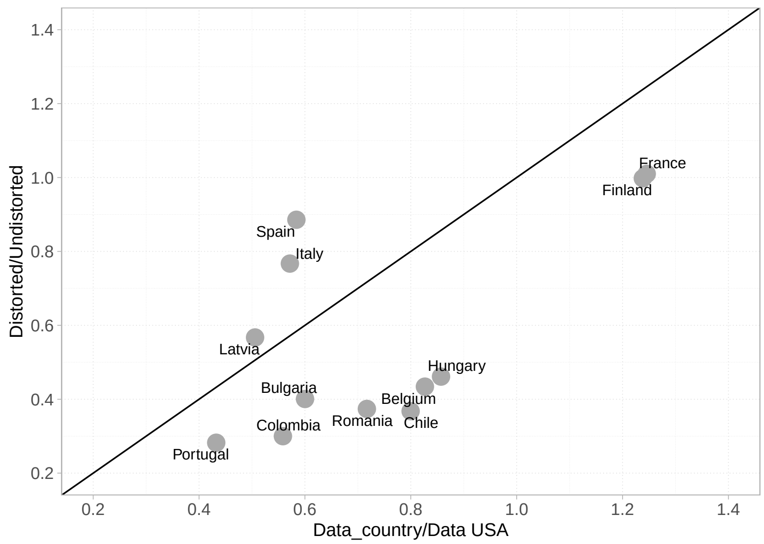


Note: According to the model, the total cost of entry in units of labor is $EC = f_e\,(1+\tau^e)$. To get at a model equivalent entry cost from the Doing Business Indicator (World-Bank 2019), we proceed as follows. We start adding the total cost of starting a firm and acquiring electricity as a proportion of income per capita from the Doing Business Database, $DB = \frac{(start+electricity)}{(Y/L)}$. Multiplying the DBI by the inverse of the labor share, which we take from the Penn World Table version 9.0 (Feenstra et al. 2015, Zeileis 2019), we get the level of the cost of entry in units of labor: $DB^L = \frac{(start+electricity)}{(Y/L)} * \frac{Y}{W*L} = \frac{(start+electricity)}{W}$. Expressed in this fashion, the Doing Business's cost of entry is comparable to $f_e\tau^E$ in the model. Thus, to isolate the Doing Business counterpart of τ^E, we divide DB^L by the calibrated value of the technological component of the cost of entry, f_e (i.e., $\tau^E_{DB} = \frac{DB^L}{f_e}$). The vertical axis of the figure plots $log\left(1+\tau^E_{DB}\right)$, the de jure proxy of the entry barrier, against $log\left(1+\tau^E\right)$ in the horizontal axis, the model-based entry.

model tracks the DBI very closely in advanced economies like France, Finland, and Italy, whereas it identifies notably larger barriers in the middle- to low-income range. Ghana is an exception of a low-income country where the entry barrier in the model is roughly identical to the DBI, while Spain stands out among advanced economies in the model by backing out an entry subsidy.

Are the differences in magnitude across measures of entry barriers economically significant? The next section addresses the macroeconomic consequences and shows that regulation-based entry barriers largely underestimate the productivity losses associated with entry taxation. Here, we focus on the micro-level implications of this measure, highlighting its shortcoming in accounting for cross-country differences in average firm size. Concretely, we solve for each country's distorted stationary equilibrium inputing entry barriers from the DBI, and assess the ability of the resulting distribution of average firm size to account for the data. The productivity elasticity of distortions γ_i remains the same as before.

Figure 6: Average Firm Size under the Doing Business Indicator's Measures of Barriers to Entry



Note: The figure plots the average firm size in each country's stationary equilibrium under distortion pairs $\left\{\tau_i^{E,DBI}, \gamma_i\right\}$. $\tau_i^{E,DBI}$ stands for the entry barriers from the World Bank's Doing Business Indicator World-Bank 2019, and γ_i denotes the productivity elasticity estimate of idiosyncratic distortions. Units in the vertical axis are reported as ratios between the distorted average firm size relative to the one in the undistorted allocation. The horizontal axis reports the average firm size in each country relative to the US.

Figure 6 shows that the DBI's measure of entry barrier largely fails to account for average firm size differences in the data. Combined with the depressing effect of idiosyncratic distortions on average firm size, the countervailing force exerted by the de jure proxy of entry barriers falls short of bringing the average firm size distribution closer to the data.

5.5 Discussion

As with any accounting exercise that identifies distortions as residuals from equilibrium conditions, it is only within the boundaries of the theory that the estimated residual constitutes a distortion. The remainder of this section discusses modeling assumptions abstractions in the theory that could cast doubt on the identification exercise's validity.

5.5.1 Validation for the Endogenous Productivity Distribution

One important channel through which distortions translate into changes in the economy's average firm size is the innovation decisions of firms and the associated stationary distribution of productivity. Biases in the properties of these distributions will transmit directly into the average firm size and therefore on the estimate of the entry barrier.

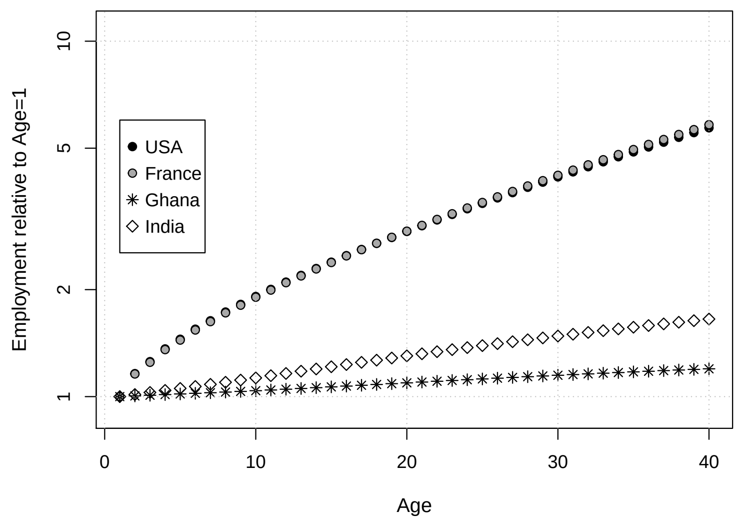
We seek reassurance in the model's prediction for the productivity distribution by exploring two sources of validation: the life cycle dynamics of firms and the share of firms at the top of the productivity distribution. Hsieh and Klenow [2014] show salient differences in the life cycle of firms between the US, India, and Mexico, which can be used to gauge the plausibility of the firms' innovation decisions in response to the mix of distortions in the economy. The share of firms with employment levels above a certain size in turn provides information about a non-targeted moment of the size distribution against which predictions of the model can be compared.

Figure 7 illustrates the model's implications for the life cycle growth of firms for a few countries at the top, middle, and bottom of the income distribution (US, France, India, and Ghana). We simulate each country's life cycle dynamics of employment from a cohort of entrants, taking prices, exit, and innovation decisions as in each country's stationary equilibrium.

The figure shows that our theory of innovation, interacted with the estimated distortions, is capable of delivering life cycle dynamics commensurable with the empirical evidence. The US and France, two countries with fairly little distortions, feature fast-growing firms upon birth. India, with more pronounced idiosyncratic distortions, exhibits a flatter life cycle growth of magnitudes in the ballpark of those documented in Hsieh and Klenow [2014], whereas Ghana, with the most severe misallocation, features an even flatter dynamic of employment growth.

Turning to the ability of the model to deliver plausible shares of large firms,

Figure 7: Life-Cycle Dynamics



Note: At each country's stationary equilibrium, we simulate a cohort of entering firms and compute the average employment of the cohort as it ages. The figure plots the average employment size of the cohort relative to the size at birth. For visual clarity, the figure reproduces the cases of the U.S. and France, two high income countries, India, for which Hsieh and Klenow [2014] provide an empirical counterpart, and Ghana, a low income country

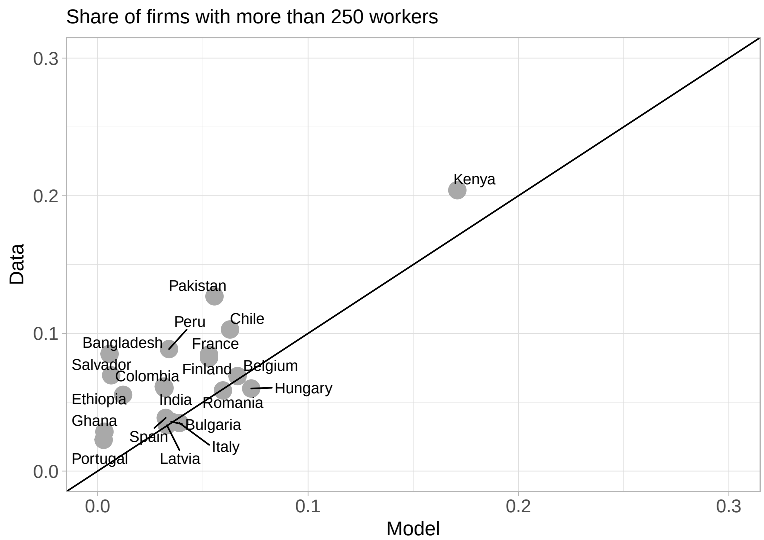
we report the share of firms with 250 workers of more both in the model and in the data. Figure 8 shows that the model does a fair job in tracking this non-targeted moment of the firm-size distribution, particularly in the highest income countries.[31]

5.5.2 Mark-Ups

An alternative interpretation of the model's inference of entry barriers is that they are picking up cross-country differences in mark-ups. Given that the un-

[31] Countries for which the model under-predicts the fraction of firms with 250 workers or more could be regarded as ones where the model also over-estimates the entry barrier. However, in these countries, the model also slightly under-predicts the share of firms below a certain size, a force which counteracts the under-prediction at the top. Essentially, the figures of the top and bottom of the size distribution reflect that log-linear productivity-dependent idiosyncratic distortions excessively reallocate employment from the top and the bottom to the middle of the distribution. We argue that despite this tendency, the model does a reasonable job at capturing the tails, and that the counteraction of the biases mutes their transmission into the estimate of the entry barrier.

Figure 8: Fraction of Firms with 250 Workers or More



Note: The figure shows the share of firm with 250 workers or more, both in the model (horizontal axis) and in the data (vertical axis). As throughout the paper, the data refer to firms with 10 workers or more.

derlying theory assumes constant and identical mark-ups across countries, it is plausible that this source of model misspecification may be biasing the inference of the entry barrier. This hypothesis gains traction in the context of a growing interest in global trends in market power (De Loecker and Eeckhout, 2018). We argue, however, that within the theoretical framework that underpins the recent estimation strategies, mark-ups manifest in the data as part of the idiosyncratic distortions. For instance, under the Cobb-Douglas production and assuming mark-ups are the sole distortion, De Loecker and Warzynski, 2012 show that the markup is closely related to the ratio of total expenditure on variable inputs over sales. This ratio, however, is exactly picking up the wedge to the firm's first-order condition that underlies the idiosyncratic distortion.[32] Similarly, considering Edmond et al., 2018 as an example of a theory of endogenous mark-ups, mark-ups generate a departure from the otherwise log-

[32]This can be seen by working from the firm's first-order condition. Letting $\mu = \frac{\theta}{\theta - 1}$ be the constant markup implied by the model, it follows from the first-order conditions that $\frac{P_i Q_i}{w L_i} = \frac{\mu}{(1-\tau)}$, where τ is the idiosyncratic distortion. Thus, it follows that variation in the ratio of sales to variable cost (given by only labor in our model) is explained by the idiosyncratic distortion.

linear relationship between firm-level employment and productivity, increasing employment for relative low levels of productivity and reducing it relative to the constant mark-up baseline for more productive firms, which is exactly the kind of deviation induced by a productivity-dependent idiosyncratic distortion profile. It may be that entry barriers generate a particular distribution of mark-ups across firms, but from the inference point of view, this distribution will be reflected in the idiosyncratic distortions, which we are taking into account at the time of identifying the entry barrier.[33]

5.5.3 Other Considerations

We address a number of other potential concerns in Appendix C. These concerns range from 1) country-specific interest rates to 2) cross-country differences in wages due to exogenous TFP differences to 3) cross-country differences in production structures induced by cross-country differences in comparative advantage across sectors. In all cases, we show that the baseline estimates of entry barriers are unchanged. While we do not provide a formal treatment, the appendix also discusses the role of the informal sector and the possibility of biases in the entry barrier due to cross-country differences in the productivity distribution of entrants.

5.6 Entry Barriers, Allocative Distortions, and TFP

Having identified the pair of distortions that reconcile the theory with cross-country differences in the firm size distribution, we now proceed to evaluate the macroeconomic consequences of these distortions. As a reminder, TFP in the economy is given by

$$TFP = M^{\frac{1}{\theta-1}} \frac{\left(\widetilde{\Omega}^w\right)^{\frac{\theta}{\theta-1}}}{\widetilde{\Omega}} * \frac{L_p}{L},$$

[33] Peters [2019] studies an environment where the distribution of mark-ups responds endogenously to changes in the costs of creating firms and launching new products.

where $\widetilde{\Omega}^w$ and $\widetilde{\Omega}$ are two summary statistics of the cross-sectional distribution of productivity, given by

$$\widetilde{\Omega}^w = \int e^{\omega} \left(1 - \tau_\omega\right)^{\theta - 1} d\widetilde{M}\left(\omega\right)$$

$$\widetilde{\Omega} = \int e^{\omega} \left(1 - \tau_\omega\right)^{\theta} d\widetilde{M}\left(\omega\right).$$

In this section we are interested in the following: 1) quantifying the productivity gains from removing entry barriers and idiosyncratic distortions, 2) decomposing the gains into those accruing from each type of distortion, 3) characterizing the role of the interaction between distortions in shaping the aggregate effects, and 4) comparing the gains between the model-based and the regulation-based estimates of barriers to entry.

5.6.1 TFP Gains from Removing Distortions

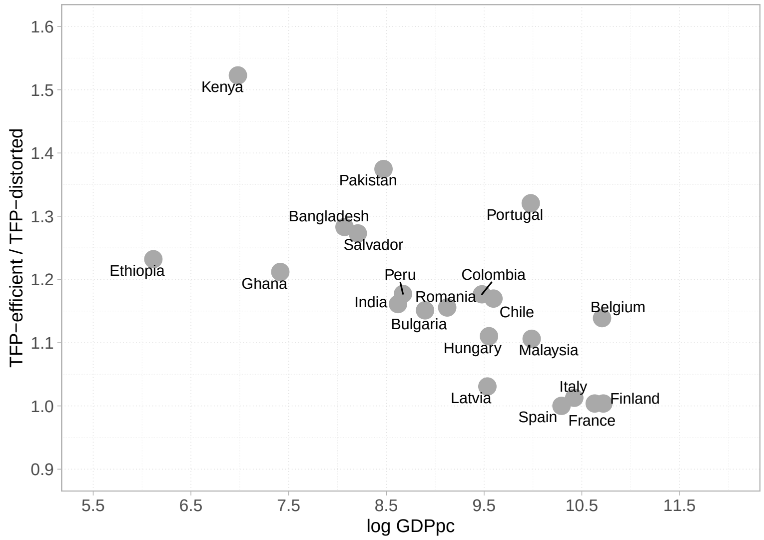
We begin by quantifying the gains in aggregate productivity that would accrue in each country if all distortions were removed. We illustrate in Figure 9 the TFP in the undistorted allocation $\{0, 0\}$ relative to the distorted one for each country's pair of distortions $\left\{\tau_i^E, \gamma_i\right\}$. We find productivity gains that are sizable in the least developed economies, reaching up to 50 percent and decreasing with income per capita until becoming roughly equal to zero in advanced economies.

We further investigate the extent to which these gains can account for the observed differences in productivity across countries. To this end, we normalize the TFP gains in the model by the observed TFP gap with respect to the US in the data. That is, we compute

$$\text{TFP gap closed} = \frac{\left(\frac{TFP\{0,0\}}{TFP\left\{\tau_i^E, \gamma_i\right\}}\right)^{Model}}{\left(\frac{TFP^{USA}}{TFP_i}\right)^{Data}},$$

where $\left(\frac{TFP\{0,0\}}{TFP\left\{\tau_i^E, \gamma_i\right\}}\right)^{Model}$ is the productivity gain from removing $\left\{\tau_i^E, \gamma_i\right\}$ in

Figure 9: TFP Gains Removing All Distortions



Note: The figure reports the gains in TFP resulting from dismantling τ_i^E and γ_i in each country and achieving the undistorted allocation. These gains are reported the log of GDP per capita, taken from Penn World Table version 9.0 for 2014 (Feenstra et al. 2015Zeileis 2019).

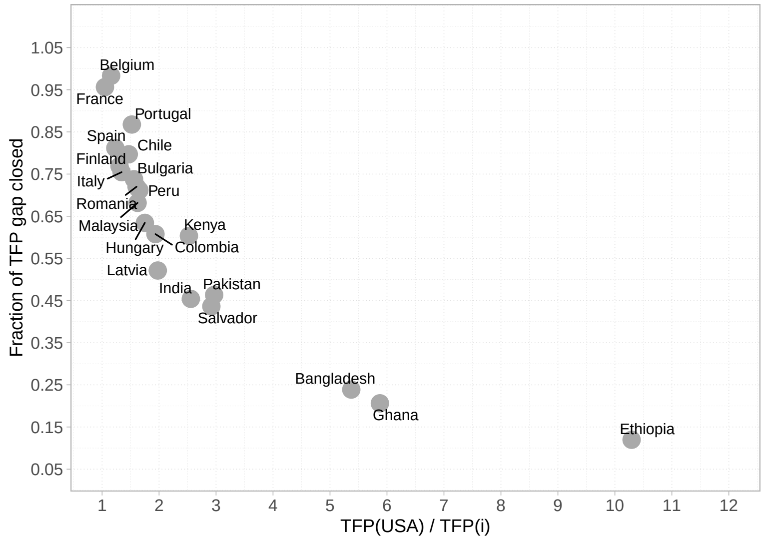
the model, plotted in Figure 9, and $\left(\frac{TFP^{USA}}{TFP_i}\right)^{Data}$ denotes the TFP gain between the US and each country in the data.[34]

We see from Figure 10 that distortions account for a notable share of the observed TFP gaps in the data. For instance, the combined effect of distortions accounts for more than 40 percent of the gaps in countries with less than half of the US' TFP. The distortions in our model, however, can account for only 20 percent of the gaps in countries like Ghana and Ethiopia, for which TFP ranges between one sixth and one-tenth of TFP in the US.

Decomposition The theory can be used to decompose the total productivity gains into those stemming from dismantling allocative distortions and those from reversing entry barriers. We show the results of this decomposition in Figure 11. As reference, the left panel of the figure reproduces the total gains.

[34]TFP gaps in the data are taken from $ctfp$ in Penn World Table version 9.0 for the year 2014 (Feenstra et al. 2015, Zeileis 2019), with the exception of Ethiopia, Ghana, and El Salvador, for which $ctfp$ is missing. In these cases, TFP was computed as $\frac{Y}{K^{1/3}L^{2/3}}$, using $rgdpna$ as the real GDP, $rkna$ as the real capital stock, and emp as the labor input.

Figure 10: Accounting for TFP Gaps in the Data



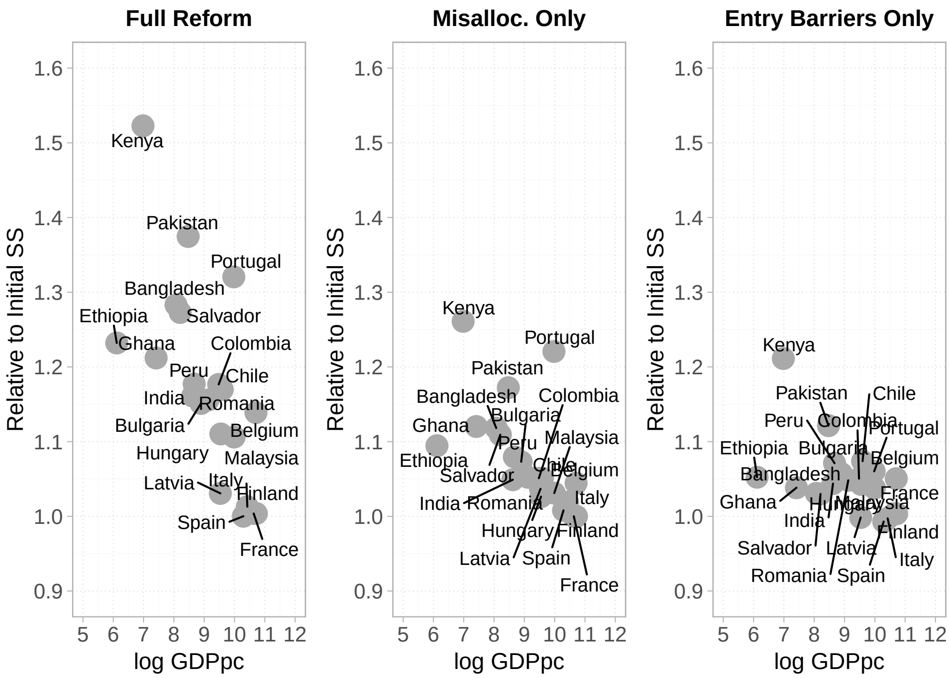
Note: The figure shows the TFP gains from dismantling τ_i^E and γ_i in the model relative to the productivity gap between the US and each country's TFP in the data. Formally, we plot TFP gapclosed = $\frac{\left(\frac{TFP\{0,0\}}{TFP\{\tau_i^E,\gamma_i\}}\right)^{Model}}{\left(\frac{TFP^{USA}}{TFP_i}\right)^{Data}}$ against the observed gaps; $\left(\frac{TFP^{USA}}{TFP_i}\right)^{Data}$. TFP in the data is read from $ctfp$ in Penn World Table version 9.0 for the year 2014, with the exception of Bangladesh, Ethiopia, Ghana, El Salvador, and Pakistan for which $ctfp$ is missing. In these cases, TFP was computed as $\frac{Y}{K^{1/3}L^{2/3}}$, using $rgdpna$ as the real GDP, $rkna$ as the real capital stock, and emp as the labor input.

The middle panel shows the gains from only removing allocative distortions, which we accomplish by setting $\gamma = 0$ while holding the entry tax rate fixed at the estimated level. The right panel reports the gains from removing the entry barriers, setting $\tau^E = 0$ and preserving the idiosyncratic distortion at the estimated value of γ.

Figure 11 reveals that allocative distortions play a slightly more prominent role in the least developed economies, whereas the gains are more evenly distributed between allocative distortions and entry barriers in the most advanced ones. The noteworthy exception among the latter group is Portugal, where misallocation accounts for a larger share of the gains.

A second implication from Figure 11 is that the total gains are larger than the sum of the individual gains. Proposition 3 established that when both distortions interact, they mitigate each other's effect on aggregate productivity. It follows, then, that the difference between the total gains and the sum of

Figure 11: Contribution of Entry and Allocative Distortion to TFP Gains

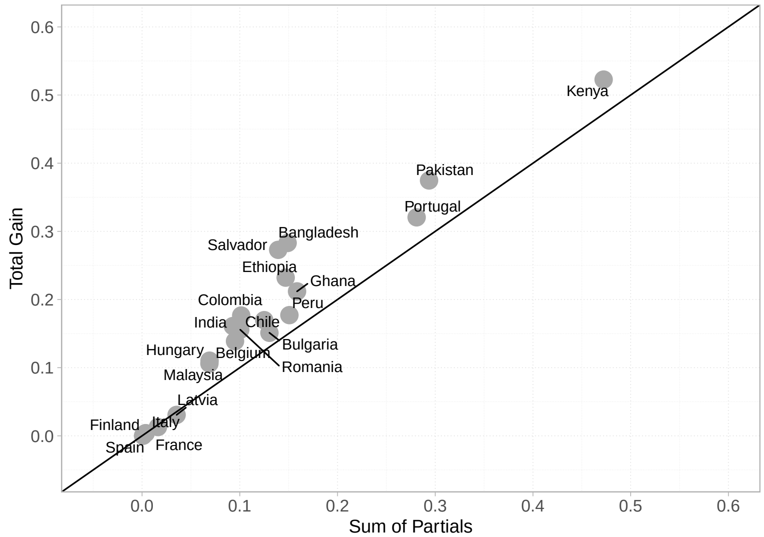


Note: The figure decomposes the total gains in TFP resulting from the withdrawal of both entry barriers and idiosyncratic distortions into those stemming from removing a single distortion while holding the other one fixed. The left-most panel reproduces the overall gains. The middle panel computes the gains from removing idiosyncratic distortions, setting $\gamma_i = 0$ while keeping the corresponding entry barriers τ_i^E. The right-most panel sets the entry barrier to zero, $\tau_i^E = 0$, keeping the productivity elasticity of distortions γ_i fixed.

the partial gains captures the mitigating force of the interaction. To better appreciate the quantitative significance of this mitigation, Figure 12 illustrates the total gains in TFP on the vertical axis against the sum of the partials on the horizontal axis and a 45-degree line.

Figure 12 shows that the mitigation is substantial in countries where idiosyncratic distortions and entry barriers are both prevalent, reaching up to 15 percent in the case of El Salvador, whereas it is close to zero in advanced economies where entry barriers are virtually nonexistent. As a byproduct, Figure 12 raises a word of caution on interpreting the magnitudes from studies that evaluate the aggregate effects of particular distortions in isolation. Not accounting for interactions with other frictions may significantly overestimate

Figure 12: Role of the Interaction: Total TFP Gains versus Sum of Partial Reforms



Note: The figure illustrates the overall TFP gains from removing entry barriers and idiosyncratic distortions relative to the sum of the gains from removing one distortion at the time while holding the remaining one fixed. The 45-degree line is added for reference to identify the magnitude of the mitigation effect of the coexistence of distortions.

its aggregate effect.

Macroeconomic Consequences of Entry Barriers under De Jure-Based Estimates We conclude the quantitative analysis by resuming the discussion on the biases incurred when measuring entry barriers with regulation-based indicators. We showed in Section 5.4 that these estimates, combined with idiosyncratic distortions, largely fail to account for the observed cross-country differences in average firm size. Here, we assess the potential for an underestimation of the macroeconomic implications.

Formally, our experiment consists of quantifying the differential TFP gains that result from withdrawing model-based entry barriers relative to those stemming from removing the World Bank's DBI's measure of entry taxation. Since, as we showed, the interaction between distortions exerts a nontrivial effect on the magnitudes of partial liberalizations, we compute these differentials both in the context of an active interaction, preserving the underlying idiosyncratic distortions, and in a context with no misallocation, as in the majority of existing studies on entry regulation.

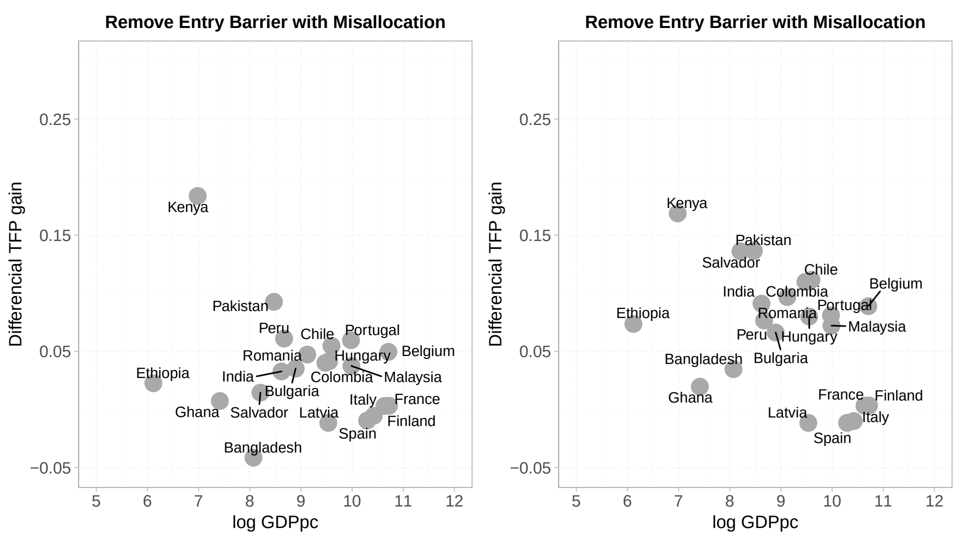
Concretely, letting $\tau_i^{E,DBI}$ denote the entry barrier in country i according to the DBI, we compute the following two TFP differentials:

$$\frac{TFP\left\{0,\gamma_i\right\}}{TFP\left\{\tau_i^E,\gamma_i\right\}}-\frac{TFP\left\{0,\gamma_i\right\}}{TFP\left\{\tau_i^{E,DBI},\gamma_i\right\}}$$

$$\frac{TFP\left\{0,0\right\}}{TFP\left\{\tau_i^E,0\right\}}-\frac{TFP\left\{0,0\right\}}{TFP\left\{\tau_i^{E,DBI},0\right\}}.$$

The productivity elasticity of distortions γ_i is unchanged and remains to be given by the WLS estimate of the regression defined in equation 18.

Figure 13: TFP Gains: Model-Based versus World Bank's Doing Business Indicator Measures of Entry Barriers



Note: The left panel of the figure shows the differential TFP gain between removing entry barriers measured as in the model relative to entry barriers imputed from the World Bank's Doing Business Indicator (WorldBank 2019). Formally, the left panel illustrates $\frac{TFP\{0,\gamma_i\}}{TFP\{\tau_i^E,\gamma_i\}}-\frac{TFP\{0,\gamma_i\}}{TFP\left\{\tau_i^{E,DBI},\gamma_i\right\}}$ in the vertical axis, against the log of GDP per capita in the horizontal one. GDP per capita is drawn from Penn World Table version 9.0, for the year 2014. The right panel in turn computes the differential TFP gain from removing each type of entry barrier assuming no underlying misallocation (i.e.,).

As shown in the left panel of Figure 13 , the DBI underestimates the TFP gains by an average of 4 percent in the case of underlying misallocation,[35] and an average of 8 percent when the interaction is shutdown. Notice in turn that these are just the average differentials. Naturally, our model-based measure does not add much in countries where entry barriers are low, regardless of how we measure them. However, the productivity differential can be as high as 15 to 20 percent in the least developed economies.

As a final attempt to emphasize the quantitative importance of these productivity gain differentials, we turn to decomposition of the total TFP gains into those coming from the idiosyncratic distortions and from the entry barriers. We performed this decomposition in Section 5.6.1 and concluded that the model-based entry barriers play a similar role than idiosyncratic distortions in shaping the aggregate gains. Here we reassess the relative contribution of each distortion by applying the same decomposition as in Section 5.6.1 but assume the entry barriers are given by the DBI (i.e., we assume the distortion pair is given by $\left\{\gamma_i, \tau_i^{E,DBI}\right\}$). The results are reported in Figure 14.

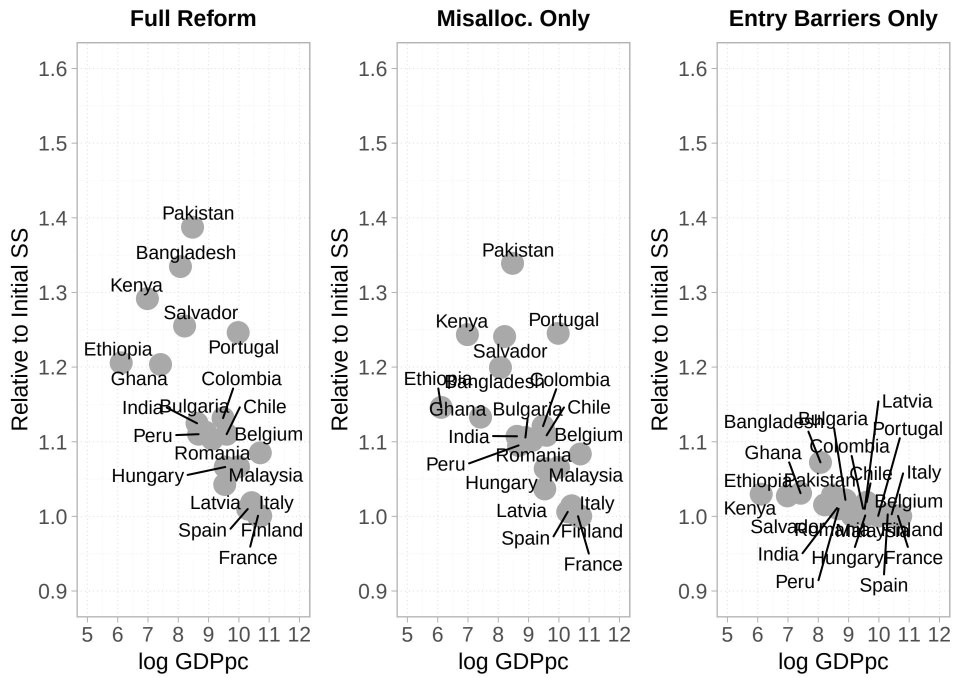
It follows from Figure 14 that a regulation-based indicator such as the DBI significantly underestimates the relative importance of entry barriers in explaining cross-country differences in aggregate productivity. The TFP gains attributable to this measure of entry barrier are not only lower, on average, below 5 percent, but it is also more homogeneous across countries relative to what we found in Figure 11 for the model-based estimate.

6 Conclusion

Studying the interaction between entry barriers and idiosyncratic distortions in the context of a standard model of firm dynamics, we proposed and implemented a strategy to measure these distortions from firm-level data and quantify their implications for total factor productivity The key distinguishing feature of the methodology is the use of cross-country differences in average firm size to discipline the inference of distortions. Exploiting the countervail-

[35]The negative differential corresponds to a case where the model identifies a negative entry barrier that, as stated earlier, is ruled out as a possibility in the DBI.

Figure 14: Contribution of Regulation-Based Entry Barriers and Idiosyncratic Distortions to TFP Gains



Note: The figure decomposes the total gains in TFP resulting from the withdrawal of both entry barriers and idiosyncratic distortions into those stemming from removing a single distortion while holding the other one fixed. Entry barriers are taken from the World Bank's Doing Business Indicator (World-Bank 2019). The left-most panel reproduces the overall gains. The middle panel computes the gains from removing idiosyncratic distortions, setting $\gamma_i = 0$ while keeping the corresponding entry barriers τ_i^E. The right-most panel sets the entry barrier to zero, $\tau_i^E = 0$, keeping the productivity elasticity of distortions γ_i fixed.

ing forces exerted by each type of distortion on the average firm size, and the independence identifying idiosyncratic distortions from equilibrium variables, the methodology exploits the equilibrium conditions of our model of the firm size distribution to infer the entry tax rate that rationalizes a country's ratio of average firm size with respect to the US under the measured misallocation.

We identified sizable entry barriers around the world. In developed economies, such as France, Spain, and Italy, the model-based entry barriers are closely captured by the World Bank's DBI, a prominently used regulation-based indicator of barriers to entry. In middle- and low-income countries, however, the model- and regulation-based estimate start to diverge, finding significantly higher entry barriers in the model. The differences between the model- and regulation-based estimate have notable micro and macro implications. We showed that not only is the DBI unable to replicate the cross-country differences in the average size distribution, but it also underestimates the TFP gains associated with their removal by between 4 to 8 percent.

A limitation of our study is its inability to pinpoint any concrete distortion underlying both the idiosyncratic distortions and the non-regulatory component of entry barriers. Our goal here was to re-spark interest in a type of distortion that was previously deemed weak in explaining cross-country differences in income. We leave a more thorough investigation of the drivers of entry taxes for future research.

References

U.S. Small Business Administration. Firm Size Data. 2015. URL https://www.sba.gov/node/12162.

Andrew Atkeson and Ariel Tomas Burstein. Innovation, firm dynamics, and international trade. *Journal of Political Economy*, 118 (3):433–484, 06 2010. URL http://ideas.repec.org/a/ucp/jpolec/v118y2010i3p433-484.html.

Pierre Bachas, Roberto N. Fattal Jaef, and Anders Jensen. Size-dependent tax enforcement and compliance: Global evidence and aggregate implications. *Journal of Development Economics*, 140:203 – 222, 2019. ISSN 0304-3878. doi: https://doi.org/10.1016/j.jdeveco.2019.05.001. URL http://www.sciencedirect.com/science/article/pii/S0304387818308447.

Levon Barseghyan and Riccardo DiCecio. Entry costs, industry structure, and cross-country income and TFP differences. *Journal of Economic Theory*, 146 (5):1828–1851, September 2011. URL https://ideas.repec.org/a/eee/jetheo/v146y2011i5p1828-1851.html.

Pedro Bento and Diego Restuccia. Misallocation, Establishment Size, and Productivity. *American Economic Journal: Macroeconomics*, 9(3):

267–303, July 2017. URL https://ideas.repec.org/a/aea/aejmac/v9y2017i3p267-303.html.

Hernan Moscoso Boedo and Toshihiko Mukoyama. Evaluating the effects of entry regulations and firing costs on international income differences. *Journal of Economic Growth*, 17(2):143–170, June 2012. doi: 10.1007/s10887-006-9007-6. URL https://ideas.repec.org/a/kap/jecgro/v17y2012i2p143-170.html.

Francisco J. Buera and Roberto N. Fattal-Jaef. The dynamics of development : innovation and reallocation . Policy Research Working Paper WPS 8505, World Bank Group, 2018. URL http://documents.worldbank.org/curated/en/273571530327069374/pdf/WPS8505.pdf.

Francisco J. Buera, Joseph P. Kaboski, and Yongseok Shin. Finance and development: A tale of two sectors. *American Economic Review*, 101(5): 1964–2002, August 2011.

Jan De Loecker and Jan Eeckhout. Global market power. Working Paper 24768, National Bureau of Economic Research, June 2018. URL http://www.nber.org/papers/w24768.

Jan De Loecker and Frederic Warzynski. Markups and firm-level export status. *American Economic Review*, 102(6):2437–71, May 2012. doi: 10.1257/aer.102.6.2437. URL http://www.aeaweb.org/articles?id=10.1257/aer.102.6.2437.

Pablo N. D'Erasmo and Hernan J. Moscoso Boedo. Financial structure, informality and development. *Journal of Monetary Economics*, 59(3):286 – 302, 2012. ISSN 0304-3932. doi: https://doi.org/10.1016/j.jmoneco.2012.03.003. URL http://www.sciencedirect.com/science/article/pii/S0304393212000372.

Simeon Djankov, Rafael La Porta, Florencio Lopez-de Silanes, and Andrei Shleifer. The Regulation of Entry*. *The Quarterly Journal of Economics*, 117(1):1–37, 02 2002. ISSN 0033-5533. doi: 10.1162/003355302753399436. URL https://doi.org/10.1162/003355302753399436.

Chris Edmond, Virgiliu Midrigan, and Daniel Yi Xu. How costly are markups? Working Paper 24800, National Bureau of Economic Research, July 2018. URL http://www.nber.org/papers/w24800.

Roberto N. Fattal-Jaef. Entry and Exit, Multiproduct Firms, and Allocative Distortions. *American Economic Journal: Macroeconomics*, 10 (2):86–112, April 2018. URL https://ideas.repec.org/a/aea/aejmac/v10y2018i2p86-112.html.

Robert C. Feenstra, Robert Inklaar, and Marcel P. Timmer. The next generation of the penn world table. *American Economic Review*, 105(10):3150–3182, 2015. URL http://www.ggdc.net/pwt/.

FranÃ§ois Gourio and Nicolas Roys. Size-dependent regulations, firm size distribution, and reallocation. *Quantitative Economics*, 5(2):377–416, 2014. doi: 10.3982/QE338. URL https://onlinelibrary.wiley.com/doi/abs/10.3982/QE338.

Nezih Guner, Gustavo Ventura, and Xu Yi. Macroeconomic implications of size-dependent policies. *Review of Economic Dynamics*, 11(4):721–744, October 2008. URL http://ideas.repec.org/a/red/issued/07-73.html.

Berthold Herrendorf and Arilton Teixeira. Barriers to entry and development. *International Economic Review*, 52(2):573–602, 2011. URL https://EconPapers.repec.org/RePEc:ier:iecrev:v:52:y:2011:i:2:p:573-602.

Hugo Hopenhayn and Richard Rogerson. Job turnover and policy evaluation: A general equilibrium analysis. *Journal of Political Economy*, 101 (5):915–38, October 1993. URL http://ideas.repec.org/a/ucp/jpolec/v101y1993i5p915-38.html.

Chang-Tai Hsieh and Peter J. Klenow. Misallocation and Manufacturing TFP in China and India. Discussion Papers 07-006, Stanford Institute for Economic Policy Research, July 2007. URL https://ideas.repec.org/p/sip/dpaper/07-006.html.

Chang-Tai Hsieh and Peter J. Klenow. Misallocation and manufacturing tfp in china and india. *The Quarterly Journal of Economics*, 124(4):1403–1448, November 2009. URL http://ideas.repec.org/a/tpr/qjecon/v124y2009i4p1403-1448.html.

Chang-Tai Hsieh and Peter J. Klenow. The Life Cycle of Plants in India and Mexico. *The Quarterly Journal of Economics*, 129 (3):1035–1084, 2014. URL https://ideas.repec.org/a/oup/qjecon/v129y2014i3p1035-1084.html.

Erzo G. Luttmer. Models of Growth and Firm Heterogeneity. *Annual Review of Economics, Vol. 2, pp. 547-576, 2010*, 2010. doi: 10.1146/annurev.economics.102308.124410.

Virgiliu Midrigan and Daniel Yi Xu. Finance and misallocation: Evidence from plant-level data. *American Economic Review*, 104(2):422–58, February 2014. doi: 10.1257/aer.104.2.422. URL http://www.aeaweb.org/articles?id=10.1257/aer.104.2.422.

Alexander Monge-Naranjo, Juan M. SÃ¡nchez, and RaÃŒl SantaeulÃ lia-Llopis. Natural resources and global misallocation. *American Economic Journal: Macroeconomics*, 11(2):79–126, April 2019. doi: 10.1257/mac.20170381. URL http://www.aeaweb.org/articles?id=10.1257/mac.20170381.

Michael Peters. Heterogeneous markups, growth, and endogenous misallocation. *Econometrica*, forthcoming, 2019.

Markus Poschke. The regulation of entry and aggregate productivity*. *The Economic Journal*, 120(549):1175–1200, 2010. doi: 10.1111/j.1468-0297.

2010.02367.x. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1468-0297.2010.02367.x.

Markus Poschke. The Firm Size Distribution across Countries and Skill-Biased Change in Entrepreneurial Technology. IZA Discussion Papers 7991, Institute for the Study of Labor (IZA), February 2014. URL https://ideas.repec.org/p/iza/izadps/dp7991.html.

Diego Restuccia and Richard Rogerson. Policy distortions and aggregate productivity with heterogeneous establishments. *Review of Economic Dynamics*, 11(4):707 – 720, 2008. ISSN 1094-2025. doi: DOI:10.1016/j.red.2008.05.002. URL http://www.sciencedirect.com/science/article/B6WWT-4SHVT18-1/2/b41c6321273abafbd4503b0868544319.

World-Bank. *Doing Business 2019*. Number 30438 in World Bank Publications. The World Bank, June 2019. URL https://www.doingbusiness.org/.

Achim Zeileis. *pwt9: Penn World Table (Version 9.x)*, 2019. URL https://CRAN.R-project.org/package=pwt9. R package version 9.1-0.

A Technical Appendix

Here we provide the proofs for propositions 1, 2, and 3.

The equilibrium of the simple model boils down to a free-entry condition and a labor market clearing condition. The unknowns are given by the ratio of aggregates that shape individual decisions, $\frac{Y}{w^\theta}$, and the number of firms, M. The ratio of aggregates can be solved directly from the free-entry condition given its independence from knowledge about the number of firms.[36] Recall that the free-entry condition, from equation 20, is given by

$$f_e\left(1+\tau^e\right)=\frac{(\theta-1)^{\theta-1}}{\theta^\theta}\frac{Q}{w^\theta}\left[\frac{1}{\left(\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1\right)}\right],$$

which equates the entry cost with the value of entrant in units of labor. It follows from here that we can solve for the aggregates as

$$\frac{Y}{w^\theta}=f_e\left(1+\tau^E\right)\left(\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1\right)\frac{\theta^\theta}{(\theta-1)^{\theta-1}}.$$

Average labor demand in turn is given by

$$\widehat{L_p}=\left(\frac{\theta-1}{\theta}\right)^\theta\frac{Y}{w^\theta}\widetilde{\Omega} \tag{23}$$

$$\widetilde{\Omega}=\frac{\delta}{\mu}\left[\frac{1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]. \tag{24}$$

Plugging in the definition of $\frac{Y}{w^\theta}$, we get

$$\widehat{L_p}=f_e\left(1+\tau^E\right)(\theta-1)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]. \tag{25}$$

[36] Notice the simplification allowed for by the exogeneity of exit, which turns the right hand side of the free-entry condition, the expected value of an entrant, is independent from any other equilibrium object besides $\frac{Y}{w^\theta}$.

Labor market clearing in turn requires that

$$L = M * f_e \left\{ \left(1+\tau^E\right)(\theta-1)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]+\delta\right\},$$

where we are using the steady state result from the law of motion for the number of firms that $M_e = M * \delta$, implying that total labor demand due to entry costs becomes $M*\delta*f_e$. Solving for the number of firms, and normalizing the population to $L = 1$, we get

$$M = \frac{1}{f_e\left\{(1+\tau^E)(\theta-1)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]+\delta\right\}}. \tag{26}$$

A.1 Ratio of Number of Firms

All of the propositions involve characterizing the ratio of the number of firms between pairs of equilibrium allocations, given by equations 14 and 15 in the main paper.

Taking the ratio of the number of firms between an economy with distortions $\{\tau^e, \gamma\}$ and an economy with misallocation only, $\{0, \gamma\}$, we get

$$\frac{M\left(\tau^E,\gamma\right)}{M\left(0,\gamma\right)} = \frac{f_e\left\{(\theta-1)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]+\delta\right\}}{f_e\left\{(1+\tau^E)(\theta-1)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]+\delta\right\}},$$

which we can rewrite as

$$\frac{M\left(\tau^E,\gamma\right)}{M\left(0,\gamma\right)} = \frac{(\theta-1)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]}{\left\{(1+\tau^E)(\theta-1)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]+\delta\right\}} + \frac{\delta}{\left\{(1+\tau^E)(\theta-1)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]+\delta\right\}}.$$

Multiplying and dividing the first term on the right hand side by the average labor demand in the economy with misallocation and entry barriers, we get

$$\frac{M\left(\tau^{E},\gamma\right)}{M\left(0,\gamma\right)}=\frac{\frac{(\theta-1)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]}{(1+\tau^{E})(\theta-1)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]}\frac{\left(1+\tau^{E}\right)(\theta-1)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]}{\left\{(1+\tau^{E})(\theta-1)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]+\delta\right\}}}{+\frac{\delta}{\left\{(1+\tau^{E})(\theta-1)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]+\delta\right\}}}.$$

The first term of the right hand side is now the outcome of the product of the ratio of the average size between the misallocation-only economy and the multiple distortion economy,

$$\frac{\widehat{L_p}\left(0,\gamma\right)}{\widehat{L_p}\left(\tau^{e},\gamma\right)}=\frac{\left(\theta-1\right)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]}{\left(1+\tau^{E}\right)\left(\theta-1\right)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]},$$

and the share of production labor in total labor demand in the multiple distortion economy, given by

$$\omega_{L_p}=\frac{\left(1+\tau^{E}\right)\left(\theta-1\right)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]}{\left\{\left(1+\tau^{E}\right)\left(\theta-1\right)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]+\delta\right\}}. \tag{27}$$

Thus, the ratio of the number of firms can now be expressed as

$$\frac{M\left(\tau^{E},\gamma\right)}{M\left(0,\gamma\right)}=\frac{\widehat{L_p}\left(0,\gamma\right)}{\widehat{L_p}\left(\tau^{e},\gamma\right)}*\omega_{L_p}+\omega_{M_e},$$

where we denote with $\omega_{M_e}$the share of labor demand due to entry on total employment, given by

$$\omega_{M_e}=\frac{\delta}{\left\{\left(1+\tau^{E}\right)\left(\theta-1\right)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]+\delta\right\}}.$$

Finally, noting that $\omega_{L_p} + \omega_{M_e} = 1$, and substituting back in the ratio of the number of firms, we get

$$\frac{M\left(\tau^{E},\gamma\right)}{M\left(0,\gamma\right)} = 1 + \left[\frac{\widehat{L_p}\left(0,\gamma\right)}{\widehat{L_p}\left(\tau^{E},\gamma\right)} - 1\right]\omega_{L_p}, \tag{28}$$

which, not surprisingly, is identical to 14. A similar logic can be followed to derive equation 15:

$$\frac{M\left(\tau^{E},\gamma\right)}{M\left(\tau^{E},0\right)} = 1 + \left[\frac{\widehat{L_p}\left(\tau^{E},0\right)}{\widehat{L_p}\left(\tau^{E},\gamma\right)} - 1\right]\omega_{L_p}. \tag{29}$$

A.2 Proof of Proposition 1

The result in proposition 1 follows directly from taking the ratio between average sizes across allocations and making use of formula 25. Doing so we get

$$\frac{\widehat{L_p}\left(\tau^{e},\gamma\right)}{\widehat{L_p}\left(0,\gamma\right)} = \frac{f_e\left(1+\tau^{E}\right)\left(\theta-1\right)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]}{f_e\left(\theta-1\right)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]}$$

$$\iff$$

$$\frac{\widehat{L_p}\left(\tau^{e},\gamma\right)}{\widehat{L_p}\left(0,\gamma\right)} = \left(1+\tau^{E}\right). \tag{30}$$

Q.E.D.

A.3 Proof of Proposition 2

Similarly, taking the ratio of the average size between the economies with $\{\tau^{e},\gamma\}$ and $\{\tau^{e},0\}$, we get

$$\frac{\widehat{L_p}\left(\tau^{E},\gamma\right)}{\widehat{L_p}\left(\tau^{E},0\right)} = \frac{\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]}{\left[\frac{\frac{\delta+\rho}{\mu}-1}{\frac{\delta}{\mu}-1}\right]}. \tag{31}$$

Showing that this ratio is < 1 boils down to

$$\frac{\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]}{\left[\frac{\frac{\delta+\rho}{\mu}-1}{\frac{\delta}{\mu}-1}\right]} < 1$$

$$\Longleftrightarrow$$

$$\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta+\rho}{\mu}-1}\right] < \left[\frac{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}-1}\right]$$

$$\Longleftrightarrow$$

$$\frac{\delta+\rho}{\mu}\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma\frac{\delta}{\mu}-\frac{\delta}{\mu}-\frac{\delta+\rho}{\mu}-\frac{\theta}{\theta-1}\gamma+1 < \frac{\delta}{\mu}\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma\frac{\delta+\rho}{\mu}-\frac{\delta+\rho}{\mu}-\frac{\delta}{\mu}-\frac{\theta}{\theta-1}\gamma+1.$$

Canceling terms,

$$\frac{\delta}{\mu} < \frac{\delta+\rho}{\mu},$$

which is true iif $\rho > 0$.

Q.E.D.

A.4 Proof of Proposition 3

Our definition of the interaction between distortions is given by the sign of the derivative of the ratio of the number of firms with respect to the value of the underlying distortion. For instance, considering the role of allocative distortions in shaping the strength of the decline in the number of firms as a result of entry barriers, the interaction with respect to the underlying misallocation is given by $\frac{\partial\left[\frac{\widetilde{M}\left(\tau^E,\gamma\right)}{\widetilde{M}(0,\gamma)}\right]}{\partial\gamma}$. Similarly, the interaction of the change in the number of firms due to misallocation with respect to the underlying entry tax rates is given by $\frac{\partial\left[\frac{\widetilde{M}\left(\tau^E,\gamma\right)}{\widetilde{M}(\tau^E,0)}\right]}{\partial\tau^E}$.

Exploiting the result that the ratio of average sizes are independent from the underlying distortion, it follows from equations 28 and 29 that the direction of the interaction will be given by the change in the production employment share with respect to the underlying distortion under study. Formally,

$$\frac{\partial\left[\frac{\widetilde{M}\left(\tau^E,\gamma\right)}{\widetilde{M}\left(\tau^E,0\right)}\right]}{\partial\tau^E}=0+\left[\frac{AvSize\left(\tau^E,0\right)}{AvSize\left(\tau^E,\gamma\right)}-1\right]*\frac{\partial\omega_{L_p}}{\partial\tau^E} \tag{32}$$

$$\frac{\partial\left[\frac{\widetilde{M}\left(\tau^E,\gamma\right)}{\widetilde{M}\left(0,\gamma\right)}\right]}{\partial\gamma}=0+\left[\frac{AvSize\left(0,\gamma\right)}{AvSize\left(\tau^E,\gamma\right)}-1\right]*\frac{\partial\omega_{L_p}}{\partial\gamma}. \tag{33}$$

Solving for the derivatives of the labor share defined in 27 we get:

$$\frac{\partial\omega_{L_p}}{\partial\tau^E}=\frac{\frac{\delta}{\mu}\left(\theta-1\right)\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]\delta}{\left[\left(1+\tau^E\right)\left(\theta-1\right)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]+\delta\right]^2}>0$$

$$\frac{\partial\left(\omega_{L_p}\right)}{\partial\gamma}=\frac{\delta}{\left\{\left(1+\tau^E\right)\left(\theta-1\right)\frac{\delta}{\mu}\left[\frac{\frac{\delta+\rho}{\mu}+\frac{\theta}{\theta-1}\gamma-1}{\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1}\right]+\delta\right\}^2}*\left[\left(1+\tau^E\right)\left(\theta-1\right)\frac{\delta}{\mu}\frac{\theta}{\theta-1}\left(\frac{-\frac{\rho}{\mu}}{\left(\frac{\delta}{\mu}+\frac{\theta}{\theta-1}\gamma-1\right)^2}\right)\right]<0.$$

It follows that the production employment share rises with entry barriers and decreases with misallocation. Taking this into account in the equations for the interaction, 32 and 33, and recalling that

$$\frac{AvSize\left(\tau^E,0\right)}{AvSize\left(\tau^E,\gamma\right)}\geq 1$$

$$\frac{AvSize\left(0,\gamma\right)}{AvSize\left(\tau^E,\gamma\right)}\leq 1,$$

it follows immediately that

$$\frac{\partial\left[\frac{\widetilde{M}\left(\tau^E,\gamma\right)}{\widetilde{M}\left(\tau^E,0\right)}\right]}{\partial\tau^E}\geq 0$$

$$\frac{\partial \left[\frac{\widetilde{M}\left(\tau^{E},\gamma\right)}{\widetilde{M}(0,\gamma)} \right]}{\partial \gamma} \geq 0.$$

Q.E.D.

B Data Appendix

Table 2 lists the countries in our sample and provides a brief description of the data sources.

Table 2: Firm-Level Databases: List of Countries and Data Sources

Country	**Database**	**Description**
Ethiopia	Central Statistical Agency (CSA): Large and Medium Scale Manufacturing and Electricity Industries Survey, 2011	Census of firms employing more than 10 workers
Ghana	Ghanaian Statistical Service (GSS), National Industrial Census, 2003	Census of more than 10 workers
Kenya	Kenya National Bureau of Statistics (KNBS)- Census of Industrial Sector, 2010	Census of all formal firms
Bulgaria, Belgium, Finland, France, Hungary, Italy, Latvia, Portugal, Romania, Spain	Amadeus, 2014	(see note below for selection criteria)
Chile	ENIA (Encuesta Nacional Industrial Annual), 2013	Census of all industrial firms with 10 or more workers
El Salvador	EAM (Encuesta Anual Manufacturera), 2004	Census of all industrial firms with 10 or more workers
India	ASI (Annual Survey of Industry), 2004–2005	Census of all industrial firms with 10 or more workers in case of power usage; 20 or more workers without power usage
Colombia	EAM (Encuesta Anual Manufacturera) 2016	Census of all industrial firms with 10 or more workers
Malaysia	Census of Manufacturing Sector, 2015	Census of all industrial firms with no size cutoff
Peru	National Economic Census, 2008	Census of all firms with no size cutoff but restricts analysis to 10 workers or more
Bangladesh	Survey of Manufacturing Industries, 2012	Census of large firms, representative sample of all firms with 10 workers or more. Stratification by size class and 4-digit industry, sampling weights provided
Pakistan	Census of Manufacturing Industries, 2005	Census of registered manufacturing firms

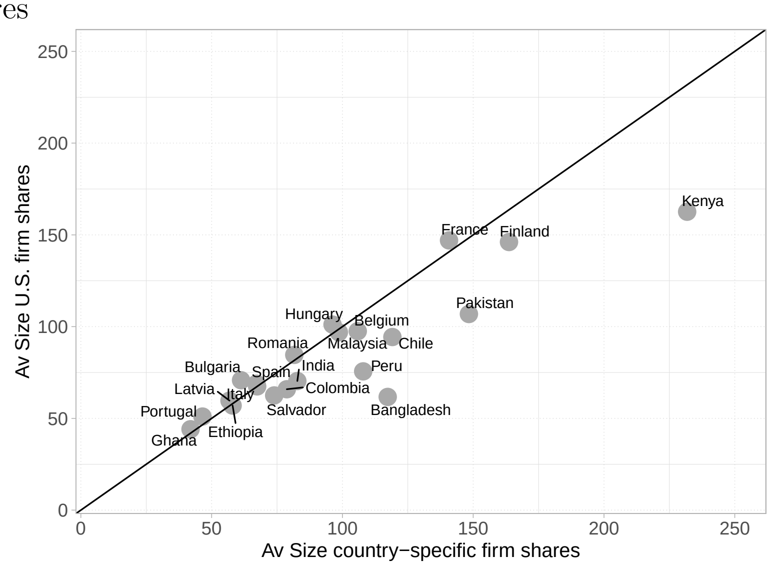
Note: Selection criteria for Amadeus database is as follows. Keep in sample if 1) the ratio of aggregate manufacturing employment in Amadeus to aggregate manufacturing employment in Eurostat is greater than or equal to 80 percent, and 2) the ratio of manufacturing employment between Amadeus and Eurostat in each bin of Eurostat's size distribution above the worker threshold (10–20, 20–50, 50–250, 250+) is also greater than or equal to 80 percent.

The average size in the US manufacturing sector is read from the Business Dynamics Statistics database for the year 2007. As we did with the rest of the countries, we computed average firm size in the US and truncated the size distribution at ten workers or more.The data on the share of firms across two-digit manufacturing industries in the US stems from the Small Business Administration database , also for the year 2007.

B.1 Average Firm Size: Country-Specific versus Fixed Distribution of Firms across Sectors

Figure 15 shows the extent to which controlling for cross-country differences in the distribution of firms across two-digit manufacturing industries affects the computation of the average firm size.

Figure 15: Average Firm Size: Fixed versus Country-Specific Distribution of Firm Shares



Note: The horizontal axis measures the average firm size, conditional on ten-plus workers, according to each countries distribution of firms across two-digit industries. The vertical axis measures the average size conditional on ten-plus workers, applying the US' distribution of firms across two-digit industries to all countries. The list of countries in the sample is reported in Table 2.

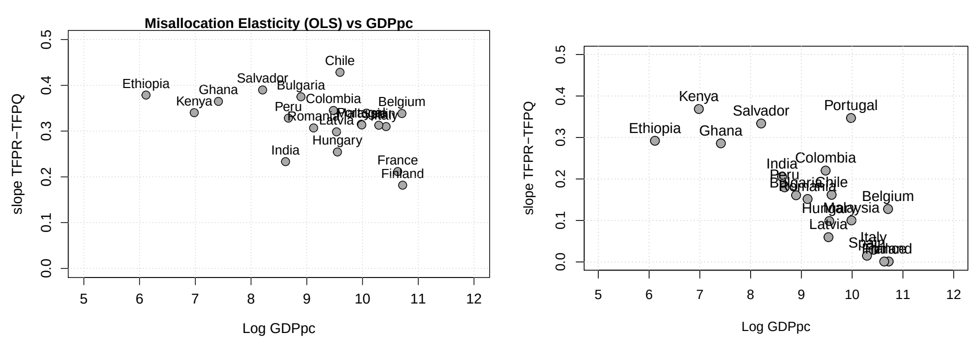
The overall pattern in figure is that controlling for production structures, by fixing the distribution of firms across 2-digit industries to the U.S. one, either reduces the average firm size relative to adopting each country's distribution or leaves it roughly unchanged. This is important for it implies that, had we not control for production structures, the entry barriers that attain the empirical average firm size distribution would have been at least higher than the ones we infer in the baseline.

B.2 OLS versus WLS Estimates of the Productivity Elasticity of Distortions

Using employment shares as weights, we implement a WLS estimation of the regression coefficient between $log\,(TFPR)$ and $log\,(TFPQ)$. The intuition is that if there indeed are a few really productive firms in the economy and they are less affected by idiosyncratic distortions, then the weighting of observations by their size will help reflect this feature in the overall estimate of the distortion slope. This in turn will translate into weaker incentives in high-income countries to cut down on innovation and a higher share of firms in the right tail of the productivity distribution.

Figure 16 reports the regression coefficients against income per capita under OLS (left panel) side-by-side with the estimates under WLS (right panel).

Figure 16: TFPR – TFPQ Regression Coefficients: Baseline versus Weighted Least Squares



Note: The left panel illustrates the regression coefficient between $log\,(TFPR)$ and $log\,(TFPQ)$in the current manuscript, estimated under ordinary least squares, and its relationship with the log of real income per capita. The right panel illustrates the same information under a weighted least squares estimation, using the employment share of the firm as weight.

As can be seen from the figure, the WLS regressions yield significantly lower values of the distortion elasticity in high-income countries (France, Finland, Spain, Belgium) and roughly identical values in poor ones relative to the OLS estimates. Since the regression coefficients are reported as differences with the US' elasticity, we see from the right-most panel that France, Finland, Italy, and Spain feature patterns of idiosyncratic distortions that are virtually identical to that of the US. Some middle-income countries like Chile and Colombia also face a lower value when estimated under WLS, whereas low-income countries like Salvador, Kenya, Ethiopia, India, and Peru face a less notable change.

C Robustness Appendix

In this section we present a series of extensions to the model in order to assess the results' robustness.

C.1 Country-Specific Rates of Return to Capital

The baseline analysis in the paper assumes a common discount factor across countries, thereby imposing a common rate of return to capital. Proposition 2, however, establishes that the discount factor participates in the determination of the elasticity with which the average firm size in the economy falls in response to productivity-dependent idiosyncratic distortions. Thus, our estimate of entry barriers may be biased by the constant discount factor assumption.

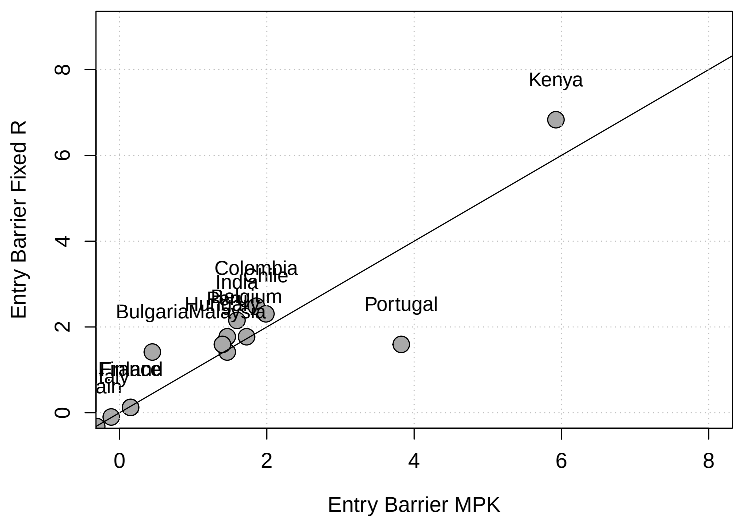
To gauge the extent to which relaxing this assumption matters for the results, we compute country-specific rates of return to capital, i.e., the marginal product of capital, and then calibrate country-specific discount factors as

$$\beta_i^{MPK} = \frac{1}{(1 + MPK_i - \delta)}.$$

The marginal products of capital are computed as in Monge-Naranjo et al. 2019, which provides a novel approach to accurately measure the land shares in the economy.

We re-estimate the model-based entry barriers under this calibration strategy and compare the outcome with the baseline estimates. We show the results in Figure 17, which illustrates the entry barrier under a fixed discount factor in the vertical axis (the baseline) against the country-specific one in the horizontal axis. Figure 17 shows that the entry barriers identified under calibrated

Figure 17: Entry Barriers in the Model: Fixed Interest Rate versus Calibrated Marginal Products of Capital



rates of return to capital are very similar to the ones identified under a fixed interest rate across countries. Portugal shows the biggest divergence due to the fact that we find a marginal product of capital that, after depreciation, gives an almost zero rate of return to capital.

C.2 Higher Wages in Higher-Income Countries and Biases in the Identification of the Entry Barrier

Another source of concern may be that cross-country differences in the wage rate could interfere with the inference of the entry barrier in the model. The concern is justified in that the entry costs in the model are denominated in units of labor. Thus, even though the amount of labor required to start a firm is assumed fixed across countries, differences in the cost may emerge from differences in the wage rate.

Suppose countries differed not only in their business environment, which we capture here with the entry barriers and idiosyncratic distortions, but also in an exogenously given TFP component. This exogenous component will generate differences in wages. However, it follows directly from the analysis of the simple model that the average firm size in the economy is independent from any aggregate force that does not distort the relative average profitability between potential entrants and incumbents. Aggregate differences in TFP indeed raise the entry cost but also raise the present value of profit in the same proportion, so the expected profits of entrants and the average profits of incumbents remain unchanged and so does the average firm size.

To see this formally in the context of the simple model, the free-entry condition and the average firm size equations are given by

$$f_e\left(1+\tau^e\right)=\frac{(\theta-1)^{\theta-1}}{\theta^\theta}\frac{Y}{w^\theta}\widetilde{\Omega}_e$$

$$\widehat{L_p}=\left(\frac{\theta-1}{\theta}\right)^\theta\frac{Y}{w^\theta}\widetilde{\Omega}.$$

An exogenous TFP term would lift $\widetilde{\Omega}_e$ (expected profits of entrants) and $\widetilde{\Omega}$ (average profits of incumbents) in identical proportions. From the free-entry condition, $\widetilde{\Omega}_e$ going up means that $\frac{Y}{w^\theta}$ has to fall to rebalance the free-entry condition. Because $\widetilde{\Omega}$ rose by the same amount as $\widetilde{\Omega}_e$, the decline in aggregates also leaves the average firm size $\widehat{L_p}$ unchanged.

More generally, insofar as the cross-country differences in the wage rate stem from forces that leave the expected and average profitability ratios unchanged, these differences will be immaterial to the average firm size and therefore to the magnitude of the entry barrier.

C.3 Comparative Advantage and Aggregation of Average Size across Sectors

A related concern is that sectoral differences in TFP across manufacturing industries may create a bias in the identification of the entry barrier even if we control for cross-country differences in firm share across these industries. The bias may arise if these sectoral differences in productivity, in addition to attracting firms to the industry, which we control for in our analysis, also affect the average size of the industry itself. Thus, while we would be controlling for Chile allocating a larger share of firms to copper-related industries, we would not be controlling for the average firm size in such industry being higher in Chile than in the US.

We address this comment by extending the simple model of Section 3 to a two-sector version, introducing exogenous differences in TFP across sectors, and characterizing the determination of the average firm size in the stationary equilibrium. We show that the independence of the average firm size to exogenous TFP differences also applies to the two-sector version. That is, assuming sectors 1 and 2 differ in an aggregate TFP component A_1 and A_2, the average firm size of sectors 1 and 2 are independent from A_1 and A_2. This implies that, while sectoral differences in productivity do determine the share of firms in each industry, their average firm sizes are independent from these differences, and hence the concern raised at the beginning is attenuated.

The model's barebones is as follows. Production is given by a nested CES structure of goods 1 and 2, each of which is produced from a bundle of a continuum of heterogeneous varieties; i.e.,

$$Y = \left[\alpha_1 Y_1^{\frac{\lambda-1}{\lambda}} + \alpha_2 Y_2^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}} \tag{34}$$

$$Y_i = \left[\int \left[y_i^d(\omega)\right]^{\frac{\theta-1}{\theta}} M_i f_i(e^\omega)\, de^\omega\right]^{\frac{\theta}{\theta-1}}, \tag{35}$$

where $\lambda < \theta$. The number of firms in each sector is denoted with M_i, and $f_i(e^\omega)$ stands for the cross-sectional distribution of productivity that, assuming the same deterministic growth of idiosyncratic productivity and the exogenous exit of the simple model in Section 3, inherits a Pareto shape.

Assuming perfectly competitive markets in each sector's good Y_i and in the final good Y, it follows that the demand function for any given variety of good i is given by

$$p_1(\omega) = P_i^{\frac{\theta-\lambda}{\theta}} Y^{\frac{1}{\theta}} \alpha_i^{\frac{\lambda}{\theta}} y_i(\omega)^{\frac{-1}{\theta}}.$$

Notice that $\theta = \lambda$ and we are essentially back in the one-sector model. As is familiar, the price index for good i is given by

$$P_i = \left[\int p_i(\omega)^{1-\theta} M_i f_i(e^{\omega}) de^{\omega} \right]^{\frac{1}{1-\theta}}.$$

The final good is the numeraire.

Production of each variety results from the following production functions

$$y_i(\omega) = (A_i)^{\frac{1}{\theta-1}} (e^{\omega})^{\frac{1}{\theta-1}} l(\omega),$$

where A_i is the sector-wide productivity component and, as before, e^{ω} indexes idiosyncratic productivity. We preserve the same profile of idiosyncratic distortions given by $[1-\tau_{\omega}] = (e^{\omega})^{-\frac{\gamma}{\theta-1}}$, where γ defines the distortion elasticity with respect to $log(TFPQ)$.

There is free entry into each sector upon payment of labor-denominated entry cost f_e, assumed constant across sectors. Entrants in each sector draw productivity from a common distribution $\Gamma(\omega)$.[37]

Solving for the profit maximization problem of producers of intermediate varieties under monopolistic competition, and working out the algebra as in the one-sector model, yields the following expression for the free-entry conditions:

$$f_e = \frac{(\theta-1)^{\theta-1}}{\theta^{\theta}} \frac{Y}{w^{\theta}} P_1^{\theta-\lambda} A_1 \alpha_1^{\lambda} \widetilde{\Omega}_e^1 \tag{36}$$

[37]Entry costs and entrant's productivity distribution can be made heterogeneous across sectors. However, the identification of entry barriers requires that we still make them fixed across countries.

$$f_e = \frac{(\theta-1)^{\theta-1}}{\theta^\theta} \frac{Y}{w^\theta} P_2^{\theta-\lambda} A_2 \alpha_2^\lambda \widetilde{\Omega}_e^2, \tag{37}$$

and the following expressions for the average firm sizes:

$$\widetilde{L_{p,1}} = \left(\frac{\theta-1}{\theta}\right)^\theta \frac{Y}{w^\theta} P_1^{\theta-\lambda} A_1 \alpha_1^\lambda \widetilde{\Omega}^1 \tag{38}$$

$$\widetilde{L_{p,2}} = \left(\frac{\theta-1}{\theta}\right)^\theta \frac{Y}{w^\theta} P_2^{\theta-\lambda} A_2 \alpha_2^\lambda \widetilde{\Omega}^2. \tag{39}$$

Labor market clearing in turn requires that

$$L = \left[M_1 * \widetilde{L_{p,1}} + M_2 * \widetilde{L_{p,2}} + f_e \left[\delta_1 M_1 + \delta_2 M_2\right]\right].$$

Lemma. *Let $\widetilde{\Omega}^i$ be the average productivity across incumbents in sector i, $\widetilde{\Omega}_e^i$ be the expected productivity of entrants, δ_i be the exogenous exit rate, and γ_i be the deterministic productivity growth rate. Then the average firm size in sector i is independent from A_i:*

$$\widetilde{L_{p,i}} = (\theta-1) f_e \frac{\delta_i}{\mu_i} \frac{\widetilde{\Omega}^i}{\widetilde{\Omega}_e^i}.$$

Proof. Solve for $\frac{Y}{w^\theta} P_1^{\theta-\lambda}$ and $\frac{Y}{w^\theta} P_2^{\theta-\lambda}$ from the free-entry conditions and replace in the average firm size expressions. □

It follows that the cross-country differences in average firm size are independent from comparative advantage. As in the one-sector model, forces that differ across sector, but are neural to the expect and average profitability ratios in each sector, are immaterial to the sector's average firm size.

One can continue with the characterization of the equilibrium to show that comparative advantage *does* determines the fraction of firms in each sector. That is, the ratios $\frac{M_1}{M_1+M_2}$ and $\frac{M_2}{M_1+M_2}$ are a function of $\frac{A_1}{A_2}$. Combined with lemma C.3, it follows that cross-country differences in sectoral productivity do not contaminate the sectoral average firm size and only affect the economy-wide average firm size through the distribution of firm shares across sectors.

This means, then, that our strategy of controlling for production structures by fixing the firm shares across two-digit manufacturing sectors in the US. is the theoretically correct approach and does not bias the identification of entry barriers.

C.4 Other Considerations

C.4.1 Productivity Distribution of Entrants

Unlike the stationary distribution, the distribution of productivity at entry is a primitive of the model that is assumed to be constant across countries. Heterogeneity in this distribution would feed into our estimate of the entry barrier.

One could imagine various ways of relaxing this assumption but not many ways of doing so in a disciplined fashion. For instance, one could calibrate a country-specific distribution of productivity at entry from the size distribution of entrants. This information, together with the idiosyncratic distortions of entering firms, allows us to back out a distribution of productivity from firms that are one years old. A more sophisticated approach would be to give entrants a choice of various initial distributions to draw from, at different costs. The biggest hurdle to both approaches is that most databases do not report the firm's age, making it impossible to separately identify technology adoption at entry from the innovation decisions of firms over the life cycle. Since we do count with some cross-country data on the life cycle dynamics of firms, we decided to generate country-specific stationary distributions of productivity through a technology for innovation post-entry.

One can still hypothesize about the direction in which a country-specific productivity distribution of entrants could bias the estimate of the entry barrier. We argue that it would most likely be in the direction of making the entry barrier even higher. Suppose there are two distributions to draw from, one with a higher average productivity and one with a higher entry cost than the other. In the undistorted economy, most entry would stem from the higher productivity distribution. Now assume that there are productivity-dependent idiosyncratic distortions. The economy will respond by shifting entry toward the low average productivity distribution, making (all else equal) the distorted average size be even lower than it would have, had the choice of a distribution at entry not been available.[38] As a result, a higher entry barrier would be needed to achieve a certain average size.

C.4.2 Informality and the Inference of Entry Barriers.

Another fair concern relates to the role of the informal sector. Should the average firm size that informs the inference of the entry barrier account for informal firms? Our view is that informality is a *consequence* of entry barriers. As such, we view the consideration of the informal sector as more important for the quantification of the productivity losses associated with these barriers, adding a magnifying force to the ones considered in the model, than for the inference of the barriers themselves. If we accounted for informal firms in the inference strategy, the average firm size that we would feed into the model would be substantially lower and so would the resulting barrier. That is, we would find that many firms would enter and the average size would be low due to the ease of starting a firm, when the reality, we argue, is the opposite: informality emerges and informal firms stay small because of the prohibitive costs of entering formal operations.

Rather, had we used data on the informal sector, we would have used it to generate a propagation mechanism in the quantification of the productivity losses from entry regulation. Along the lines of D'Erasmo and Boedo [2012], we could introduce an endogenous formal sector that expands or contracts as a function of the business environment and use it to assess how much higher the productivity losses from entry taxation would be had we accounted for it by this propagation mechanism. The inference of the entry barrier in turn would still be determined by the average firm size in the formal sector. In this sense, our abstraction from informality does not affect our claims about

[38] Indeed, Bento and Restuccia [2017] show theoretically that a reduction in entrant's average productivity is one of the main channels through which correlated distortions reduce TFP. That paper, however, does not offer any empirical counterpart of the size distribution of entrants across countries to be able to assess the empirical plausibility of the theory.

the magnitude and properties of entry taxation but would make our quantitative findings become lower bounds to what would take place in a model with endogenous informality.